A double-ratio method to measure fast, slow and reverse sap flows

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Sap velocity measurements are useful in fields ranging from plant water relations to hydrology at a variety of scales. Techniques based on pulses of heat are among the most common methods to measure sap velocity, but most lack ability to measure velocities across a wide range, including very high, very low and negative velocities (reverse flow). We propose a new method, the double-ratio method (DRM), which is robust across an unprecedented range of sap velocities and provides real-time estimates of the thermal diffusivity of wood. The DRM employs one temperature sensor upstream (proximal) and two sensors downstream (distal) to the source of heat. This facilitates several theoretical, heat-based approaches to quantifying sap velocity. We tested the DRM using whole-tree lysimetry in Eucalyptus cypellocarpa L.A.S. Johnson and found strong agreement across a wide range of velocities.

Keywords: double-ratio method, Eucalyptus cypellocarpa, heat-pulse-based technique, sap flow, sap flux, sap velocity, thermal diffusivity.

Introduction

Sap velocity measurements using heat-tracing techniques have been important means for studying plant water relations from whole plant to catchment scale in the tree physiology, forestry and hydrology communities (Wilson et al. 2001, Cermak et al. 2007, Buckley et al. 2011, 2012, Steppe et al. 2015). There are many heat-dissipation-based methods, but all have one or more weaknesses. Constant-power approaches such as the heat field deformation method (Nadezhdina et al. 1998) or Granier-style probes (Granier 1985) are inexpensive and robust but require empirical calibration (Clearwater et al. 1999) and consume large amounts of electrical power. Moreover, Granier-style probes cannot accurately measure small or negative sap velocities. A recent heat-pulse-based method, employing a single probe with finite heating duration (I-SPHP) (Lopez-Bernal et al. 2017), has advantages of low cost, simple fabrication and no requirement for correction of misalignment. However, that method cannot capture slow or negative flows, while an improved version, F-SPHP (Ren et al. 2020), broadens the measuring range to slightly negative flows but is more computationally intensive and sensitive to heat contamination. Both SPHP methods rely on a known thermal diffusivity (or conductivity) that requires independent measurements or calibration. Other heat-pulse-based methods such as the Tmax method (Cohen et al. 1981), compensation heat pulse method (CHPM) (Green and Clothier 1988), heat ratio method (HRM) (Burgess et al. 2001) and Sap/flow+ method (Vandegehuchte and Steppe 2012c) use little power and are traceable to first principles, but also have limitations. Tmax and CHPM cannot measure low or negative velocities (Green et al. 2003, Vandegehuchte and Steppe 2012c), Sap/flow+ suffers from difficulty in model identification, and the HRM fails at high sap velocities (Bleby et

In most heat-pulse-based methods, sapwood thermal diffusivity, $k$, is a crucial parameter in calculation of sap velocity (Vandegehuchte and Steppe 2012a, López-Bernal et al. 2014). An arbitrary value of $k$ is usually set or obtained from empirical functions related to sapwood moisture content ($m_c$) (Marshall 1958, Burgess et al. 2001, Vandegehuchte and Steppe 2012a). However, $k$ can change over seasons (Burgess et al. 2001, Chen et al. 2012) and even diurnally (López-Bernal et al. 2014), with uncertain consequences for the accuracy of calculated sap velocity. Velocity estimates from the Tmax method depend only on $k$ and known parameters. Therefore, if sap velocity is known to be zero, $k$ can be directly inferred and then treated as a constant until such a time that it can be re-calibrated (Burgess et al. 2001). Inverse modeling has also been used to calibrate $k$ (Chen et al. 2012, Vandegehuchte and Steppe 2012a), notwithstanding measurement uncertainties and issues with probe alignment. For example, empirical functions to infer $k$ from sapwood properties and $m_c$ could be validated at low sap velocities under 20 cm h$^{-1}$ (Vandegehuchte and Steppe 2012b); however, such procedures require knowledge of $m_c$, which may vary diurnally (López-Bernal et al. 2014). $k$ has never been estimated in vivo, in real time, at higher sap velocities.

Pearsall et al. (2014) proposed a method for extending the range of sap velocities that can be reliably measured using heat-pulse approaches. They combined the HRM and CHPM, using the HRM to detect small and negative sap velocities and the CHPM to detect high sap velocities. A limitation of that approach is the lack of a non-arbitrary method for selecting whether to use the HRM- or CHPM-derived estimate of sap velocity at a given time. Another concern is alternation between very different measurement approaches: an average over time of velocity based on the ratio of temperature rises in two sensors (HRM), or inference based on estimation of the instant at which two temperature rises are equal (CHPM). Research communities that use sap velocity measurements thus lack a single method that is at once energy efficient, objective, robust and capable of measuring negative, low and high velocities with a single measurement principle.

We developed a new and efficient algorithm, the double-ratio method (DRM), that combines the strengths of existing methods and is robust across an unprecedented range of sap velocities, from moderate negative velocities to very large positive velocities. The DRM is an extension of the HRM in which an additional temperature sensor (Probe #3) is installed distal to (i.e., ‘downstream’ from, in the direction of normal diurnal flow) both temperature sensors used in the HRM. The DRM estimates sap velocity based on the same principles as the HRM—namely, by calculating the ratios of heat-pulse-induced temperature rises measured in different probes. Two different velocity estimates are produced—one based on Probes #2 and #1 (the upstream sensor and the first downstream sensor) and another based on Probes #3 and #2 (the two downstream sensors)—and the value with the lesser intrinsic uncertainty (which is calculated based on temperature rises and probe positions) is retained. Furthermore, the presence of a third sensor also enables CHPM estimates of flow, which can be combined with DRM-based estimates to allow real time estimation of $k$ under high-velocity conditions. We tested the DRM experimentally using weighing lysimeters and examined its capability by numerical modeling. We discuss the strengths and weaknesses of the DRM in relation to other commonly used heat-pulse-based techniques.

**Materials and methods**

**Theory**

*Background: heat pulse theory and the HRM* Marshall (1958) showed that an instantaneous heat pulse at time $t = 0$ causes an increase in temperature ($\delta_i$) at both axial ($x_i$) and azimuthal ($y_i$) positions relative to the heater at time $t$ (s), with negative and positive $x_i$ values indicating positions upstream relative to the heater (generally closer, more proximal, to stem base) and downstream relative to the heater (more distal to stem base):

$$\delta_i = \frac{Q}{4\pi \rho c k t} \exp \left( -\frac{(x_i - V t)^2 + y_i^2}{4kt} \right), \quad (1)$$

where $V$ is the heat pulse velocity in the axial direction (cm h$^{-1}$), $Q$ is the total heat released per unit length of the heater (J m$^{-1}$), $\rho$ is the sapwood bulk density (including water and wood) (kg m$^{-3}$), $c$ is the sapwood heat capacity (J kg$^{-1}$ K$^{-1}$) and $k$ is the thermal diffusivity of the sapwood-water matrix (m$^2$ s$^{-1}$). Applying Eq. (1) to two probe locations, $i$ and $j$, and rearranging provides an estimate of $V$:

$$V_{ij} = \frac{2k}{x_j - x_i} \ln \left( \frac{\delta_j}{\delta_i} \right) + \frac{x_i + x_j}{2t} - \frac{y_j^2 - y_i^2}{2t (x_j - x_i)}. \quad (2)$$

In Eq. (2), $V$ is estimated from four types of quantities: a ratio of temperature increases between two probes ($\delta_j/\delta_i$), the thermal diffusivity $k$, the positions of the two probes ($x_i$, $x_j$, $y_i$, $y_j$), and the time elapsed since the heat pulse length ($t$). If the two probes are parallel to one another (i.e., at identical azimuthal positions relative to the flow axis), then $y_i = y_j$, giving

$$V_{ij} = \frac{2k}{x_j - x_i} \ln \left( \frac{\delta_j}{\delta_i} \right) + \frac{x_i + x_j}{2t}. \quad (3)$$

In the HRM, Eq. (3) is applied to two probes equidistant from the heater, one upstream (Probe #1, $i = 1$) and one downstream
Even if

\[ V_{HRM} = V_{12} = \frac{2k}{x_2 - x_1} \ln \left( \frac{\delta_2}{\delta_1} \right). \]  (4)

A key strength of the HRM is that it can distinguish both small and negative heat pulse velocities (when sap flows in reverse, i.e., in an upstream or proximal direction, sap transfers heat towards Probe #1, causing \( \delta_1 > \delta_2 \) and hence as heat pulse velocity increases. Any algorithm to

\[ \delta_1 = \delta_1 + \varepsilon_1 \text{ and } \delta_2 = \delta_2 + \varepsilon_2, \]

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are errors due to measurement noise, which can be either positive or negative, since they are presumably random and symmetrically distributed around zero). Applying these estimates to Eq. (4) gives

\[ \hat{V}_{12} = \frac{2k}{x_2 - x_1} \ln \left( \frac{\delta_2}{\delta_1} \right) = \frac{2k}{x_2 - x_1} \ln \left( \frac{\delta_2 + \varepsilon_2}{\delta_1 + \varepsilon_1} \right). \]  (5)

When \( V_{12} \) is large, \( \delta_1 \) is small (cf. Eq. (1)), and as a result the error term \( \varepsilon_1 \) becomes large relative to \( \delta_1 \). If \( \delta_1 \) is small enough, \( \delta_1 + \varepsilon_1 \) will be negative for some measurements (i.e., \( \varepsilon_1 \) will be negative and larger in magnitude than \( \delta_1 \)). Moreover, the proportion of such measurements will increase as \( \delta_1 \) decreases, and hence as heat pulse velocity increases. Any algorithm to apply Eq. (4) must disregard such measurements, because they make the operand of the logarithm negative and thus undefined. But because \( \varepsilon_1 \) is negative for the discarded points, the remaining (undiscarded) estimates of \( \delta_1 \) are positively biased, and hence the resulting estimates of \( V_{12} \) are negatively biased. This bias increases as the true velocity increases, which manifests experimentally as a ‘plateau’ or ‘ceiling’ in inferred heat pulse velocity (e.g., Figure 1), often in the vicinity of 30–50 cm h\(^{-1}\) (Pfautsch et al. 2011, Pearsall et al. 2014, Flo et al. 2019). The inferred velocity can actually decline even as true velocity continues to increase (Figure 1). Even if \( \delta_1 \) is non-negative, noise in \( \delta_1 \) at high velocities leads to the denominator of the heat ratio approaching zero, with the consequence that resulting estimates of \( V_{12} \) can fluctuate by orders of magnitude.

**Principles of the DRM** To alleviate the negative bias of the HRM at high \( V \), we propose a new method, the DRM, that uses a three-step approach to estimate \( V \). First, \( V_{12} \) is estimated as in the HRM. Second, an additional estimate of sap velocity \( V_{23} \) (Eq. (6)) is computed using the temperature rises at Probes #2 and #3, where Probe #3 is located downstream to Probe #2 at the same distance as Probe #2 is from Probe #1 (so \( x_3 = 3 \cdot x_2 \)):

\[ V_{23} = \frac{2k}{x_3 - x_2} \ln \left( \frac{\delta_3}{\delta_2} \right) + \frac{x_2 + x_3}{2t}. \]  (6)

The third step is to choose between the two estimates of velocity \( V_{12} \) and \( V_{23} \) to give a single, final estimate of \( V \) \( (V_{DRM}) \). On the grounds that measurements with less uncertainty are intrinsically preferable to those with greater uncertainty, we chose \( V_{DRM} \) as the value with the smaller intrinsic measurement uncertainty \( (\sigma_{12} \text{ or } \sigma_{23}) \):

\[ V_{DRM} = \begin{cases} V_{12} & \text{if } \sigma_{12} \leq \sigma_{23}, \\ V_{23} & \text{else}. \end{cases} \]  (7)

where \( \sigma_{12} \) and \( \sigma_{23} \) depend on the standard deviation of random noise for temperature sensors \( (\sigma_T) \) and on the estimated temperature rises at each probe \( (\delta_1, \delta_2, \text{ and } \delta_3) \), as follows:

\[ \sigma_{12} = \frac{2k\sigma_T}{x_2 - x_1} \sqrt{\frac{1}{\delta_2} + \frac{1}{\delta_1}}, \]  (8)

\[ \sigma_{23} = \frac{2k\sigma_T}{x_3 - x_2} \sqrt{\frac{1}{\delta_3} + \frac{1}{\delta_2}}. \]  (9)

When the true velocity is negative or very low, the uncertainty in \( V_{23} \) will be greater than that for \( V_{12} \); conversely, at high velocities, \( \sigma_{12} > \sigma_{23} \) and \( V_{DRM} \) will equal \( V_{23} \). This selection procedure avoids the need to specify an arbitrary value of velocity at which to switch from one estimate to the other, as is required in the approach of Pearsall et al. (2014).
Note that, if the probes are evenly spaced (i.e., \(x_3 - x_2 = x_2 - x_1\)), then \(\sigma_{12} > \sigma_{23}\) if \(\delta_3 > \delta_1\), and vice versa; that is, \(V_{12}\) is chosen if \(\delta_3 > \delta_1\), and \(V_{23}\) is chosen otherwise. An alternative approach, which we suggest for the sake of completeness but did not test here, would be to use the inverses of \(\sigma_{12}\) and \(\sigma_{23}\) as ‘weights’ to compute \(V_{DRM}\) as a weighted average of \(V_{12}\) and \(V_{23}\), thus: \(V_{DRM} = (V_{12}/\sigma_{12} + V_{23}/\sigma_{23})/(1/\sigma_{12} + 1/\sigma_{23})\).

**Identifying the optimal time window for averaging velocity estimates** In the HRM, \(V_{12}\) is computed as the average of repeated instantaneous estimates during a fixed period after the heat pulse. Burgess et al. (2001) suggest a 40-s time window, beginning 60 s after the pulse and ending at 100 s. The uncertainty-based approach of the DRM provides an alternative and objective way to identify the optimal time window: namely, by choosing the period that minimizes intrinsic uncertainty in velocity, computed as the standard error (SE) of the mean estimated velocity (based on the theoretical uncertainty in \(V_{DRM}\), not the standard deviation of a sample of instantaneous estimates):

\[
SE_{DRM} = \frac{1}{n_t} \sqrt{\sum_{t=a}^{b} \sigma_{DRM}^2(t)},
\]

where \(n_t\) is the number of time steps in the averaging window, \(\sigma_{DRM}\) is the lesser of \(\sigma_{12}\) and \(\sigma_{23}\) at each point in time, and the summation limits at right (\(a\) and \(b\)) denote the first and last steps in the averaging window. An efficient algorithm to minimize \(SE_{DRM}\) is to locate the moment of minimum \(\sigma_{DRM}\), compute the \(SE_{DRM}\) for time windows of varying width around the moment, and choose the width that gives the smallest \(SE_{DRM}\). We tested a variety of values for \(n_t\).

Note that all equations were derived from Marshall’s (1958) model (Eq. (1)) under the assumption of an instantaneous heat pulse, whereas in reality the heat pulse has a finite length of \(t_0\). We corrected for this by shifting each temperature time-course by \(-t_0/2\) before applying Eq. (1). The effect of this treatment is shown in Supporting Information Notes S1 and Figure S1 available as Supplementary data at Tree Physiology Online.

**Theoretical test of the DRM, HRM, CHPM and Tmax methods**

**Comparison of heat-pulse-based methods** To assess the theoretical viability of the DRM in comparison with other heat pulse methods (with similar probe settings), and to help optimize operational considerations such as the size and timing of averaging window(s), we used Eq. (1) to simulate time courses of temperature following a heat pulse. We compared the predicted values of \(V_{DRM}\) (Eq. (7)) with \(V_{HRM}\) (Eq. (4)), and also with two other estimates of \(V\), based, respectively, on the CHPM (Green and Clothier 1988) and the Tmax method (Cohen et al. 1981), the latter as modified by Kluitenberg and Ham (2004). Using the CHPM, heat pulse velocities are calculated at the time point \((t_{C(1,3)}\) when temperature rises for Probes #1 and #3 are equal, so that the ratio of temperature rises is unity and the logarithmic term including \(k\) equals zero, giving the velocity as

\[
V_{CHPM} = \frac{x_1 + x_3}{2C(1,3)}.
\]

In the Tmax method, sap velocity is calculated from the time at which the measured temperature rise is greatest after a given heat pulse \((t_m)\). This leads to

\[
V_{Tmax} = \sqrt{\frac{4k}{t_0} \ln \left(1 - \frac{t_0}{t_m}\right) + \frac{x^2_i}{t_m(t_m - t_0)}},
\]

where \(x_i\) is the distance of the probe \(i\) from the heater (Probe #2 is typically used because it usually has the greatest peak temperature). Note that if the true sap velocity is known to be zero, Eq. (12) can be set equal to zero and rearranged to provide an estimate of \(k\):

\[
k = \frac{x^2_i}{4t_m} \left(\frac{t_0}{t_m - t_0}\right) \left(\ln \frac{t_m}{t_m - t_0}\right)^{-1}.
\]

**Simulations** We performed two sets of simulations. In the first set, we simulated time courses of temperature rises for all three probes following a heat pulse, for sap velocities ranging from \(-10\) to \(+80\) cm h\(^{-1}\). We added Gaussian noise with a standard deviation of 0.02 K (using simulation module randn in Matlab) to each time course. The procedures for generating the synthetic data in each time course, as well as estimated sap velocity, are provided in Methods S1 available as Supplementary data at Tree Physiology Online. Each such simulation was repeated \(10^4\) times to estimate the probability distribution of each estimate of sap flux under random temperature noise. All simulations used sapwood properties given in Table S1 available as Supplementary data at Tree Physiology Online, and heat pulse parameters and probe settings given in Table S2 available as Supplementary data at Tree Physiology Online.

In the second set of simulations (Methods S2 available as Supplementary data at Tree Physiology Online), we determined the sensitivity of each method to inaccuracy in \(k\). While heat pulse methods typically assume constant \(k\), in practice \(k\) varies with \(m_c\), which may vary diurnally or seasonally in relation to changes in water potential and cycles of discharge and recharge of xylem water stores (Chen et al. 2012, López-Bernal et al. 2014). We simulated time courses of temperature rises as for the first set of simulations for sap velocities between \(-10\) to \(+80\) cm h\(^{-1}\), while assuming each of three \(m_c\) values (0.5, 1.0, 1.5 g cm\(^{-1}\)). We calculated \(k\) from \(m_c\) for a basic density \((\rho_0)\) of 0.5·10\(^3\) kg m\(^{-3}\) using the relationship given by Vandegehuchte and Steppe (2012b).
Theoretical test of internal calibration of \( k \) Because \( k \) may vary in relation to fluctuations in \( m_c \), using a constant value of \( k \) in the DRM (or HRM) may cause systematic errors. To reduce such errors, \( k \) can be estimated in vivo by combining the sap velocity estimate from the DRM with an independent estimate of \( V \), obtained from the CHPM, which does not explicitly depend on \( k \). Although the CHPM itself has limitations (see Introduction), this procedure provides useful information about the magnitude of diurnal and seasonal changes in \( k \). Setting \( V_{DRM} \) equal to \( V_{CHPM} \) under conditions of high sap velocity (so that \( V_{DRM} = V_{23} \)) and solving for \( k \) gives

\[
V = \frac{(x_3 - x_2)(x_1 - x_2)}{4c_{(1,3)} \ln \left( \frac{L}{L_2} \right)},
\] (14)

where the temperature rises \( \delta_2 \) and \( \delta_3 \) are those measured at time \( t_{c(1,3)} \). The smallest sap velocity at which Eq. (14) applies depends on the probe spacing and recording time, which limit the range of sap velocities that can be measured using the CHPM (Eq. (11)). For example, for our recording time (400 s after the heat pulse) and probe spacing (1.5 cm between Probes #1 and #3), the lowest sap velocity that could be inferred from Eq. (11) would correspond with the temperature rises of Probes #1 and #3 crossing over at 400 s after the heat pulse, giving \( V = (x_1 + x_3)/2t = (-0.75 + 2.25)/(2.400) = 1.88 \times 10^{-3} \) cm s\(^{-1} \), or 6.75 cm h\(^{-1} \). The maximum \( V \) at which Eq. (14) applies depends on \( \sigma_f \), \( Q \) and \( m_c \). Figure 2 illustrates how the maximum discernible sap velocity varies with \( m_c \) for various probe and heat parameters (Table S2 available as Supplementary data at Tree Physiology Online). The drier the sapwood, the broader the range of velocities to which Eq. (14) applies because smaller \( m_c \) leads to larger \( k \).

We estimated \( k \) as follows. First, we used Eq. (13) to estimate \( k \) prior to dawn, when sap velocity was negligible (as verified by lysimetry). We applied that value of \( k \) until \( V \) rose high enough (e.g., > 6.75 cm h\(^{-1} \)) in the early mornings for us to calculate \( k \) from Eq. (14). When \( V \) exceeded the range in which Eq. (14) is applicable (e.g., 50 cm h\(^{-1} \) in Figure 2a), we used the last estimated value of \( k \) until \( V \) fell again into a range in which Eq. (14) could be applied.

Validation of the DRM on trees

Lysimeter experiments We used whole-tree lysimetry to test the DRM, as well as the HRM, CHPM and Tmmax methods. Measurements were made at the Centre for Carbon, Water and Food at the Camden Campus of the University of Sydney in Brownlow Hill, NSW, Australia (34.03°S, 150.66°E) in 2016–17. Three Eucalyptus cypellocarpa saplings, each in a large container (0.9 m by 1.2 m by 0.9 m), were placed on lysimeters capable of weighing up to 1200 kg (50 g resolution, Mettler Toledo, Columbus, Ohio). The three saplings were originally planted as seedlings in the containers in 2011. Irrigation was adjusted so that we could measure sap velocity across a range of water availabilities. Trees were initially irrigated for 30 min (around 2.5 l min\(^{-1} \)) twice a day for a month, and then reduced to 20 min twice a day, then to 2 min six times a day, and finally without any irrigation for several days. Transpiration was calculated from weight loss measured by the weighing lysimeters. Additional details regarding the lysimetry data analysis are provided in Methods S3, available as Supplementary data at Tree Physiology Online.

Sap velocity probe construction and installation Each probe set comprises three temperature sensor probes and one heater
probe. Each temperature sensor contained one thermistor (QTI Sensing Solutions, Boise, Idaho, E320) inserted into an 18-gauge blunt tip needle. Thermistor sensing tips were coated in heat sink compound before inserting and then centered 1.5 cm from the distal end of the needle. Thermistor depths were fixed by placing a drop of low-viscosity cyanoacrylate glue in both the proximal and distal end of the needle. Each heater was made by feeding approximately 65 cm of Manganin wire (Goodfellow Group Ltd, Coraopolis, Pennsylvania, CU065822) through a 27.5-gauge hypodermic needle, leaving 3 to 5 cm of wire extending from the proximal end of the needle with the remainder (approximately 60 cm) protruding from the distal end, and then winding the wire tightly around the outside of the needle until the distal 3 cm of needle was covered with winding. Cyanoacrylate glue was then applied to the proximal section of winding to prevent unwinding, and the needle’s plastic base was removed by gripping the needle with pliers below the glued winding and bending it repeatedly at an angle of approximately 20° until it fractured and could be pulled off the remaining wire. The resulting ‘heater core’ was then dipped in heat sink compound and inserted into an 18-gauge needle. The total resistance of the heaters varied from 17 to 18.4 Ω and was insensitive to temperature between 0 and 70 °C. The sensors and heaters were soldered to extension cables, which were wired to an AM16-32B multiplexer (Campbell Scientific Inc., Logan, UT, USA) and relays, respectively. Initiation of the heat pulses and the temperature recordings were controlled by a CR850 datalogger (Campbell Scientific).

Probes were inserted radially into the sapwood and parallel to one another, with the aid of a portable drill press (Kanzawa, Miki, Japan, K-801) and a rigid guide plate made from angle iron and strapped to each tree. Temperature sensors were located at distances of 0.75 cm upstream (Probe #1, \(x_1 = -0.75\) cm), 0.75 cm downstream (Probe #2, \(x_2 = +0.75\) cm) and 2.25 cm downstream (Probe #3, \(x_3 = +2.25\) cm) from the heater probe. One set of sensors was installed on each tree approximately 30 cm above the soil surface; the diameters of the trees (#1, 2 and 3) were 8.2, 8.4 and 10.0 cm, respectively, at the height of each heater probe. A heat pulse of 7–16 s in duration was initiated every 15 or 30 min by applying 12 V across the heater probe wire, which induced a peak temperature rise of around 2 K in Probe #2. The subsequent temperature change of each probe was recorded every second for 5 to 10 min. All probe installations were wrapped with 15-cm thick polyester insulation and covered with Mylar-coated bubble wrap for insulation.

The temperature rise (\(\delta_i\)) was calculated by first computing the initial temperature of each probe as its average for 5 s before a given heat pulse. To account for drift in background temperature during the rise and decay of temperature following each heat pulse, we used the pchip function in Matlab to simulate changes in background temperature by smooth interpolation between each successive pre-heat-pulse initial temperature, and then subtracted the resulting changes in background temperature from each time course of temperature to give \(\delta_i\).

To validate the DRM with the mass flow rate (transpiration) measured by lysimeters, heat-pulse velocities (\(V, \text{cm h}^{-1}\)) were corrected for misalignment errors (see Methods S4 available as Supplementary data at Tree Physiology Online) and then converted to whole-tree sap flow (\(kg h^{-1}\)) following Burgess et al. (2001). Heat pulse velocity was first converted to sap flux density, \(V_s\) (‘sap velocity on total sapwood area basis’ in the terminology of Edwards et al. (1996)), as \(V_s = V \rho_s (c_w + m_c c_s)/(\rho_s c_s)\), where \(\rho_s\) is the basic density of wood (0.5 g cm\(^{-3}\) for the studied trees), \(c_w\) and \(c_s\) are specific heat capacities of the wood matrix (1200 J kg\(^{-1}\) K\(^{-1}\)) and sap water (4182 J kg\(^{-1}\) K\(^{-1}\)), respectively, \(m_c\) (g g\(^{-1}\)) is the water content of sapwood (1.1 for the trees) and \(\rho_s\) is the density of sap (10\(^{-3}\) kg cm\(^{-3}\)). Second, we scaled the resulting single-point estimates of sap flux to whole-tree sap flow (\(F, \text{kg h}^{-1}\)) empirically, by multiplying by estimated sapwood area (\(a, \text{cm}^2\)) and sap density \(\rho_s\), and then using direct measurements of whole-tree sap flow by lysimetry to produce an empirical calibration coefficient (\(f, \text{unitless}\)). Thus, \(F = f \rho_s V_s\). As comparisons of sap flow methods to lysimetry are best made when flows of water into or out of water storage tissues are smallest, typically during the middle of the day (Chuang et al. 2006, Buckley et al. 2011, Deng et al. 2017), we chose \(f\) for each tree to make mid-day maximum values of \(F\) match between sap flow and lysimetry (Table S1 available as Supplementary data at Tree Physiology Online presents values of \(a\) and \(f\) for each tree).

The empirical coefficient \(f\) incorporates two sources of potential divergence between point measurements of \(V_s\) and whole-tree sap flow: spatial variation in \(V_s\) (radial and/or azimuthal) and effects of drilling wounds. The fact that a single proportionality constant was sufficient to encompass these factors (\(f\) was nearly constant for each tree from February through May 2016; Table S1 available as Supplementary data at Tree Physiology Online) has two implications. First, it suggests that any radial or azimuthal variation that may have occurred in sap flux did not change systematically over time, such that our point measurements remained proportional to the true spatial mean of \(V_s\) within each tree. Second, it suggests that any wounding effects that may have occurred did not generate nonlinearity in the relationship between \(V_s\) and \(F\) in our trees. Future efforts to apply the DRM more broadly, and without the benefit of lysimetric calibration, may benefit from numerical simulations of flow in a three-sensor system to determine whether wound correction models derived for two-sensor systems in previous studies (Swanson and Whitfield...
Results

Theoretical testing

Operational procedures for DRM We calculated $V_{\text{DRM}}$ by averaging values (measured at 1 Hz) for window sizes varying between 5 and 100 s, with the central time point chosen as that at which either $\sigma_{23}$ or $\sigma_{23}$ was smallest. Monte Carlo simulations indicated that the optimal window size was inversely related to sap velocity (Figure 3). For example, SEs were smallest using a window width of 80–100 s for a true sap velocity of 30 cm h$^{-1}$, but using a width of 20–40 s when velocities were 80 cm h$^{-1}$ (Figure 3b and e). Time windows for the least biased estimates of $V$ did not overlap as indicated by the SE at $V = 80$ cm h$^{-1}$ when comparing time windows of 20 and 80 s (Figure 3e and f). This is due to the assumption of an instantaneous heat pulse in the Marshall Model, from which all heat-pulse-based methods are derived (see Discussion).

In theory, window sizes of 80 s can be used, but in practice, the quality of recorded temperature data deteriorates as temperatures level off and noise thus becomes increasingly prominent. For example, if we excluded temperature rises of less than 0.015 K (three times the standard deviation of temperature noise due to sensor resolution), sap velocities $>80$ cm h$^{-1}$ could not be discerned using a 100-s window. Optimal window sizes generally decreased as flow rates increased. To simplify, we suggest using a fixed window of 40 s for $V_{\text{DRM}}$ calculations—the same window size as recommended for the HRM.

The DRM algorithm identifies both the optimal size and timing of the averaging window. There is a distinct transition of optimal window size and timing in the range of velocities at which the intrinsic uncertainties of $V_{12}$ and $V_{23}$ are similar, which occurs at around $\sim 20$ cm h$^{-1}$. At lower sap velocities, $V_{12}$ is less uncertain than $V_{23}$ (hence $V_{\text{DRM}} = V_{12}$, Eq. (7)), giving an optimal averaging window 40 s wide and centered between 60 and 70 s (Figure 4)—very similar to the averaging window commonly used for the HRM (40 s wide and centered at 80 s, see Chen et al. (2012)). At intermediate sap velocities, $V_{23}$ is less uncertain than $V_{12}$ (hence $V_{\text{DRM}} = V_{23}$), and the optimal averaging window is centered at $\sim 200$ s—roughly when the temperature traces for Probes #2 and #3 intersect (see blue and red lines in Figure 4). Note that the velocity at which $V_{12}$ transitions to $V_{23}$ will depend on conditions and will differ with probe positions and heat pulse strength and length. For example, greater heat intensity ($Q/t_0$) will lead to a transition at greater flux.

Performance of the heat-pulse-based methods Simulation results with synthetic data are shown in Figure 5. Among all four methods, the DRM calculation of $V$ was consistently closest to the true value used to generate the synthetic data, deviating little from the true value for high (0.2% deviation at 80 cm h$^{-1}$), slow (0.5% at 20 cm h$^{-1}$) and negative sap velocities (1.6% at $-10$ cm h$^{-1}$). The coefficient of variation of estimates was also smallest for the DRM (e.g., 2% at 20 cm h$^{-1}$ and 0.5% at 80 cm h$^{-1}$). The HRM performed well at negative to slow sap velocities ($<40$ cm h$^{-1}$) but systematically underestimated true sap velocity when velocity was high. The CHPM only performed well at intermediate sap velocities (20–40 cm h$^{-1}$), yielding a widely diverging range of estimates when true sap velocities were high, low or negative. The poor performance of the HRM and CHPM at high sap velocities was caused by very small temperature rises at the upstream temperature sensor ($\delta_1$). Calculations that depend on $\delta_1$ become unreliable under such conditions (Figure S3a available as Supplementary data at Tree Physiology Online). The CHPM also performs poorly when sap velocity is low because it relies on precise determination of the time when $\delta_1 = \delta_3$: $\delta_3$ is small when sap velocity is low, leading to increased error (Figure S3b available as Supplementary data at Tree Physiology Online). The $T_{\text{max}}$ method is based on timing of peak temperature rise and is thus sensitive to noise at relatively slow sap velocities (e.g., 35% coefficient of variation at 20 cm h$^{-1}$).

CHPM and $T_{\text{max}}$ will perform better if noise reduction techniques are implemented, while HRM and DRM are less sensitive to noise. The applicable range of the HRM depends on magnitude of errors in measured temperatures (Figure S4 available as Supplementary data at Tree Physiology Online), which in turn depend on precision of the temperature sensor ($\sigma_1$) and the amount of heat generated during the heat pulse. The useful range of the HRM could be extended by increasing heat pulse duration or intensity.

Sensitivity to $k$ Among the four heat-pulse methods, the HRM was the most sensitive to uncertainty in $k$. Calculations of $V$ are directly proportional to $k$ using the HRM Eq. (4). Estimates of $V$ from the DRM are also sensitive to $k$ at low sap velocities ($<20$ cm h$^{-1}$) because $V_{\text{DRM}}$ is dominated by $V_{12}$ ($V_{\text{HRM}}$, Eq. (4)) under such conditions (Figure 6). However, when sap velocity is high, $V_{\text{DRM}}$ equals $V_{23}$, which is much less sensitive to $k$; this is because $V_{23}$ is the sum of two terms, of which only one depends directly on $k$ (Eq. (6)). Using $k$ as estimated using either Eq. (13) or Eq. (14) considerably improves the accuracy of the DRM (and HRM) at low sap velocities. The CHPM is completely insensitive to uncertainty in $k$ velocities.

Validating DRM with lysimetry Peak transpiration fluxes were 5.5, 6.0 and 7.0 kg h$^{-1}$, for Trees #1, #2 and #3, respectively, from February to May

1981, Burgess et al. 2001) are suitable for our three-sensor configuration.
Figure 3. The SEs of averaging $V_{DRM}$ over various sized windows are shown in (a) and (d) with their corresponding central time points on the x axis, (b) and (e) illustrate the distribution of SEs from 10,000 simulations in relation to the averaging window size, and the distribution of the estimation errors (%) in $V_{DRM}$ to the width of the window are shown in (c) and (f). True sap velocity was set to 30 cm h$^{-1}$ (a–c) or 80 cm h$^{-1}$ (d–f). The legend values (5 to 100) for (a) and (d) specify width of the averaging window in seconds. The blue boxes (b, c, e, f) have upper and lower edges defined by the 25th and 75th percentiles of 10,000 runs and contain mean values denoted by the red lines. The red crosses are the outliers greater than 2.7 standard deviations from the mean.

2016. Peak temperature rise of Probe #2 was around 2 K at midday and 0.6 K at night. Peak sap velocity calculated by the DRM was $>60$ cm h$^{-1}$ for Tree #1, $>80$ cm h$^{-1}$ for Tree #2 and $>100$ cm h$^{-1}$ for Tree #3.
The DRM reproduced diel patterns of sap flow measured by lysimetry under a wide range of flow conditions, from negligible flows at night to very large and fluctuating flows (see Figure 7 for sap flow, Figure 8 for heat pulse velocity of Tree #3 and Figure S5 available as Supplementary data at Tree Physiology Online for all three trees). The HRM, by contrast, was unable to capture heat pulse velocity >25 cm h\(^{-1}\) (>2 kg h\(^{-1}\), Figure 7). While better than the HRM in capturing large sap velocities (Figures 7 and 8), the CHPM also failed when flows were very fast or very slow (see Figure 8). The Tmax method failed to capture heat pulse velocities <20 cm h\(^{-1}\). To match peak velocity measured by lysimetry, scaling factors for Tmax had to be increased with time (see Table S1 available as Supplementary data at Tree Physiology Online). For example, scaling factors increased 1.3-fold from February to April, and 1.6-fold from February to May. The sensitivity of the Tmax method to noise and probe alignment requires constant adjustment of scaling factors as trees grow.

By comparison with lysimetry data, heat-based methods of measuring sap flow show a lag of ~10 min. A likely explanation is that water stored in stems and branches contributes to transpiration in the early morning—stores that are replenished via root water uptake in the late afternoon.

**Discussion**

*The DRM vs other heat-pulse-based methods*

The DRM presented here was more robust than other common heat pulse methods (HRM, CHPM and Tmax) across a wide range of sap velocities, in both theoretical and experimental tests. In theoretical tests, the DRM produced the least bias and smallest variance among the four methods. In experimental tests, the DRM accurately tracked diel trends in tree water use (as measured by lysimetry) from 0–7 kg h\(^{-1}\) or 0–100 cm h\(^{-1}\).

The applicable upper limit for the HRM can be roughly estimated from Eq. (4), given the resolution of temperature sensors and peak rise in temperature. For example, if we assume:

- that the temperature rise at the upstream probe (\(\delta_1\)) is reliable if its output is at least three times the standard deviation of temperature noise due to sensor resolution (approximately 0.005 K in our experiment), so that values of \(\delta_1\) below 0.015 K are excluded; and
- that the largest observed value of the temperature rise at the first downstream probe (\(\delta_2\)) is 1.0 K,

then with a probe spacing of 1.5 cm and \(k = 0.0025 \text{ cm}^2 \text{ s}^{-1}\), the upper limit of velocity that the HRM can estimate is

\[
V_{HRM,\text{max}} = \frac{2 \cdot 0.0025 \text{ cm}^2 \text{ s}^{-1}}{1.5 \text{ cm}} \ln \left( \frac{1 \text{ K}}{0.015 \text{ K}} \right)
\]

\[
= 0.014 \text{ cm s}^{-1} \left( \approx 50 \text{ cm h}^{-1} \right).
\]

In practice, the minimum uncertainty in \(\delta_1\) is likely larger than the sensor resolution, due to other factors (e.g., background fluctuations, drifts in temperature). This limit could be increased by reducing probe spacing or increasing the size of the heat pulse. However, closer spacing magnifies relative errors caused by imperfect alignment of probes during installation.

The CHPM was unable to detect high sap velocities because, like the HRM, it is limited by uncertainty in \(\delta_1\). The CHPM also fails at low sap velocities because the intrinsic uncertainty in the ‘crossover point’ of \(\delta_1\) and \(\delta_3\) is greater. Both the CHPM and the Tmax method are inherently incapable of detecting negative sap velocities (reverse flows). Testi and Villalobos (2009) suggested the resolution of the CHPM could be improved using an empirical calibration function (Vandegehiuche and Steppe 2012c), although this requires study- and species-specific solutions. Pearsall et al. (2014) obtained estimates of sap velocity >200 cm h\(^{-1}\) by supplementing the HRM with the CHPM. Those velocities far exceed values that we were able to detect using the CHPM (~40–50 cm h\(^{-1}\)). A possible reason for this difference is that the values reported in their study were corrected for wounding, whereas ours were not. Given that Pearsall et al. (2014) used temperature sensors and dataloggers with similar resolution to ours, and heater probes with similar resistances (~18 \(\Omega\)), this discrepancy in maximum velocities could also be due to wider probe spacing used in the present study (1.5 cm vs 1.2 cm used by Pearsall et al. 2014). To assess this possibility, we can use Eq. (1) to quantify the effect of probe spacing on the maximum temperature rise that one would expect to occur at the upstream probe (Probe #1). Suppose \(\rho = 1.08 \cdot 10^3 \text{ kg m}^{-3}\), \(c = 2.8 \cdot 10^7 \text{ J kg}^{-1} \text{ K}^{-1}\),

\[
\text{Figure 4. The time corresponding to the center of the optimal averaging window for the DRM varies with sap velocity. The thick red and blue solid lines are the theoretical times at which the temperature traces would intersect (red: } \tau = \frac{(x_2 + x_3)}{2V} \text{ when } \delta_2 = \delta_3 \text{ or } (x_1 + x_3)/2V \text{ when } \delta_1 = \delta_3 \text{ in the absence of temperature noise. The solid black line and black symbols are theoretical values calculated with DRM in the absence of random noise.}
\]
Figure 5. Boxplots showing the distributions of sap velocities ($V$, cm h$^{-1}$) calculated with the DRM, HRM, CHPM and Tmax methods from synthetic temperatures with Monte Carlo sampling of $\sigma_T = 0.02$ K, $t_0 = 10$ s and $m_c = 1.25$ g g$^{-1}$. The mean values are shown by red lines in the blue boxes, which have upper and lower edges defined by the 25th and 75th percentiles, and the red crosses are the outliers that are more than 2.7$\sigma$ from the mean. The CHPM was not given in panels (a) and (b) as it could not resolve negative or low flows.

Wider spacing has the advantage that a given absolute error of probe alignment during installation will cause a smaller error in calculated sap velocity.

The DRM has other advantages. Most importantly, the measurement principle Eq. (2) is the same across all sap velocities. Switching measurement principles, as required if HRM is used for low velocities and the CHPM for high velocities, creates logistical problems. Secondly, the DRM provides a theoretical basis for identifying the optimal time window in which to average...
estimates of sap velocity. We found an optimal window for *E. cypellocarpa* of 50–120 s after the heat pulse (Figure S6 available as Supplementary data at Tree Physiology Online, which is approximately equal to theoretically calculated values (Figure 4) and similar to the window used for the HRM (60–100 s). Thirdly, the DRM is less sensitive to noise than the CHPM, because the latter relies on a single intersection point, whereas the former averages data over a longer time period. Finally, unlike the HRM, the DRM is relatively insensitive to uncertainty in the value of $k$ when $V > 20 \text{ cm h}^{-1}$ because $V_{\text{DRM}} = V_{23}$ Eq. (6) at high sap velocities, and the term involving $k$ in Eq. (6) becomes small compared to the term that involves $t$.

**Impact of the finite heat pulse length**

Following Marshall’s original model, the assumption of an instantaneous heat pulse results in biased estimates of sap velocity in most methods. A small negative bias is evident in both the DRM- and HRM-based velocity estimates. The bias of the DRM increases from $-0.03\%$ when $V = 30 \text{ cm h}^{-1}$ to $-0.2\%$ when

![Figure 6. Uncertainties in V estimation due to k errors: true $k = 0.0024 \text{ cm}^2 \text{ s}^{-1}$ (a–d), underestimation of V with the DRM (a) and HRM (b) using constant $k = 0.0021 \text{ cm}^2 \text{ s}^{-1}$, and overestimation with the DRM (c) and HRM (d) for constant $k = 0.003 \text{ cm}^2 \text{ s}^{-1}$. Errors in the corrected velocity using $k$ estimated with the DRM are in (e). Note the underestimation at higher velocity with the HRM in (d) is a result of negative bias (see Figure 1).](https://academic.oup.com/treephys/article/41/12/2438/6294570)
$V = 80 \text{ cm h}^{-1}$ (Figure 9). A larger bias is evident using the CHPM (which underestimates velocity by 9% at 30 cm h$^{-1}$) because of its reliance on the precise time at which $\delta_1 = \delta_3$. Its high sensitivity to pulse duration ($t_0$) was also recognized in a data synthesis study by Flo et al. (2019), though the Tmax method can be modified to account for $t_0$ (Kluitenberg and Ham 2004), thus eliminating this bias. In practice, Tmax-based sap velocity estimates are highly uncertain when velocity is high, due to sensitivity to noise (Figure 5e). Shifting $t_0$ helps mitigate the impact of the finite heat pulse length across methods, resulting in negligible bias for the DRM (Figure 9).

**Misalignment correction**

Misalignment of probes is a major source of uncertainty for heat-pulse techniques. It affects all methods and can bias inferred sap velocity either positively or negatively. Burgess et al. (2001) noted two alternative scenarios that could be used in misalignment corrections in the HRM: either that Probe 1 is misaligned, or that Probe 2 is misaligned. Corrections are calculated for each case and are used to force calculated sap velocity to equal zero under conditions when the true sap velocity is known to be zero. Burgess et al. (2001) recommended using the average of the two corrections. However, though all three methods result in matched calculated and measured sap velocities under ‘zero-flow’ conditions, they do not yield consistent day-time sap velocities (Figure 10).

The HRM requires a known value of $k$ to correct errors due to probe misalignment. Typically, a constant $k$ is assumed. Burgess et al. (2001) used the Tmax method Eq. (15) to calculate $k$. This is a somewhat circular procedure because the Tmax method requires knowledge of probe locations. It would be preferable to concurrently solve for all three unknowns—$\Delta x_1$, $\Delta x_2$ and $k$—as shown by Chen et al. (2012). Such a
calibration approach may not be achievable unless constrained with assumptions or additional calculations (e.g., sap velocity independently inferred via an empirical function in \textit{Chen et al. (2012)}). We developed an alternative calibration procedure (\textit{Methods S4} available as Supplementary data at \textit{Tree Physiology Online}), in which alignment is first checked using the HRM method suggested by \textit{Burgess et al. (2001)}, the non-unique solution is then constrained by comparing velocities using the CHPM, which is independent of \( k \), and finally \( k \) is identified using the Tmax method under zero-flow conditions. When the average solution is chosen from the HRM method, sap velocities estimated using the DRM match well with those estimated using the CHPM (Figure 11). In this scenario, any misalignment is well corrected (CHPM is sensitive to misalignment) and \( k \) is appropriately estimated (DRM is sensitive to \( k \) at low flow).

The three-probe approach used by the DRM allows diagnosis of misalignment corrections. Combined with calculations based on the CHPM, it also allows for determination of both \( k \) and misalignment under moderate conditions (e.g., typical day time).

\textbf{Conclusions}

We present a new heat-pulse-based method, the DRM, to calculate sap velocity in woody plants. We demonstrate both theoretically and experimentally that the DRM can effectively measure a broader range of sap velocities than other common heat-pulse-based methods. Like an earlier approach that combined the HRM and CHPM (\textit{Pearsall et al. 2014}), the DRM computes two distinct estimates of sap velocity at each time point; however, the DRM has several advantages, including a theoretical basis for selecting between the two velocity estimates and for identifying

Figure 8. The heat pulse velocity calculated with DRM and other heat-pulse-based methods over the months for Tree 3; results for other trees are shown in Figure S5, available as Supplementary data at \textit{Tree Physiology Online}. X-axis is date in the format of dd/mm in 2016.
Figure 9. Comparison between heat-pulse-based methods without the time shift (red squares) and with the shift (blue circles) to account for finite heat pulse duration \( t_0 \). The original Tmax (Cohen et al. 1981) and the CHPM (Green and Clothier 1988) were shown without the \( t_0 \) shift in red and with the shift in blue. The modified Tmax method of Kluitenberg and Ham (2004) is shown in magenta triangles. Each calculation is averaged from 10,000 Monte Carlo simulations (at \( \sigma_T = 0.02 \) K, \( t_0 = 15 \) s, \( q = Q/t_0 = 80 \) J m\(^{-1}\) s\(^{-1}\), \( k = 0.002 \) cm\(^2\) s\(^{-1}\), \( m_c = 1.25 \) g g\(^{-1}\), \( \rho_b = 0.5 \cdot 10^3 \) kg m\(^{-3}\)).

Figure 10. Divergence in corrected velocities after misalignment corrections in the HRM method following Burgess et al. (2001). The blue points show the raw data without misalignment correction. \( \Delta x_1 = 0 \) assumes no misalignment of \( x_1 \) and only \( x_2 \) is corrected. Similarly, \( \Delta x_2 = 0 \) means that only \( x_1 \) is corrected and \( \Delta x_1 = \Delta x_2 \) is the average of the solutions when either \( x_2 \) or \( x_1 \) is corrected. Corrections were applied to make the calculated flow rate equal to zero at the time indicated with a red asterisk; we reasoned that the true flow rate was zero at that time because a large rain event had occurred in the 12 h prior, and reported flow had been steady for several hours.
the optimal times for averaging data, and a reduced sensitivity to noise at high velocities.

**Supplementary data**

Supplementary data for this article are available at *Tree Physiology* Online.

**Conflict of interest**

The authors have no conflict of interest.

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**Authors’ contributions**

T.N.B., M.A.A. and Z.D. planned, designed and coordinated the research. T.N.B. developed the DRM algorithm and designed and built the probes for the experiment. Z.D. conducted the numerical simulations, lysimetry experiment, data collection, analysis and interpretation, with assistance and feedback from T.N.B. H.K.V. and M.E.G. performed part of the experiment and data collection. Z.D. and T.N.B. wrote the manuscript. All authors edited successive drafts of the manuscript.

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