Technical Report

Partitioning changes in photosynthetic rate into contributions from different variables

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ABSTRACT

Changes in net CO₂ assimilation rate (A) are often partitioned into contributions from changes in different variables using an approach that is based on an expression from calculus: namely the definition of the exact differential of A, which states that an infinitesimal change in A(dA) is equal to the sum of infinitesimal changes in each of the underlying variables, each multiplied by the partial derivative of A with respect to the variable. Finite changes in A can thus be partitioned by integrating this sum across a finite interval. The most widely used method of estimating that integral is a coarse discrete approximation that uses partial derivatives of the natural logarithm of A rather than A itself. This yields biased and ambiguous estimates of partitioned changes in A. We present an alternative partitioning approach based on direct numerical integration of dA. The new approach does not require any partial derivatives to be computed, and it can be applied under any conditions to estimate the contributions from changes in any photosynthetic variable. We demonstrate this approach using field measurements of both seasonal and diurnal changes in assimilation rate, and we provide a spreadsheet implementing the new approach.

Key-words: mesophyll conductance; non-stomatal limitation; photosynthesis; quantitative limitations analysis; stomatal conductance; stomatal limitation; V_{cmax} .

INTRODUCTION

It is often useful to quantify the impact of changes in various factors on net CO₂ assimilation rate (A). Jones (1985) proposed an approach in which a finite change in A is partitioned into percentage contributions from changes in several underlying variables. This approach was later modified by Wilson et al. (2000) and then extended by Grassi & Magnani (2005), and the version proposed by the latter authors (hereafter the 'GM' approach) has been widely adopted in recent years (e.g. Flexas et al. 2006a,b, 2009, 2014; Galmés et al. 2007; Niinemets 2007; Galle et al. 2009, 2011; Perez-Martin et al. 2009, 2014;

Keenan *et al.* 2010a,b; Limousin *et al.* 2010; Misson *et al.* 2010; Sagardoy *et al.* 2010; Egea *et al.* 2011; Tomás *et al.* 2013).

The GM approach is based on the definition of the total differential of a multivariate function. That definition states that an infinitesimal change in a function equals the sum of infinitesimal changes in the function's variables, each multiplied by the function's partial derivative with respect to that variable. For example, if A is expressed as a function of N variables $(x_j, \text{ where } j = 1 \text{ to } N)$, then the total differential of A (dA) is

$$dA = \frac{\partial A}{\partial x_1} dx_1 + \frac{\partial A}{\partial x_2} dx_2 + \dots + \frac{\partial A}{\partial x_N} dx_N = \sum_{j=1}^{N} \frac{\partial A}{\partial x_j} dx_j, \tag{1}$$

where dx_j is an infinitesimal change in x_j (Table 1 lists symbols with descriptions and units). Integrating Eqn 1 across a finite interval would thus partition the finite change in A across that interval into components due to the variables x_j . The GM approach applies Eqn 1 to light-saturated assimilation rate $(A_{\rm max})$, with stomatal conductance to ${\rm CO}_2$ $(g_{\rm sc})$, mesophyll conductance $(g_{\rm m})$ and carboxylation capacity $(V_{\rm cmax})$ treated as variables:

$$dA_{\text{max}} = \left(\frac{\partial A_{\text{max}}}{\partial g_{\text{sc}}}\right) dg_{\text{sc}} + \left(\frac{\partial A_{\text{max}}}{\partial g_{\text{m}}}\right) dg_{\text{m}} + \left(\frac{\partial A_{\text{max}}}{\partial V_{\text{cmax}}}\right) dV_{\text{cmax}}.$$
 (2)

This expression is divided by A_{max} to express the changes in relative terms:

$$\begin{split} \frac{dA_{\max}}{A_{\max}} &= \left(\frac{\partial \ln A_{\max}}{\partial \ln g_{\text{sc}}}\right) \frac{dg_{\text{sc}}}{g_{\text{sc}}} + \left(\frac{\partial \ln A_{\max}}{\partial \ln g_{\text{m}}}\right) \frac{dg_{\text{m}}}{g_{\text{m}}} + \left(\frac{\partial \ln A_{\max}}{\partial \ln V_{\text{cmax}}}\right) \frac{dV_{\text{cmax}}}{V_{\text{cmax}}} \\ &\equiv l_{\text{s}} \frac{dg_{\text{sc}}}{g_{\text{sc}}} + l_{\text{mc}} \frac{dg_{\text{m}}}{g_{\text{m}}} + l_{\text{b}} \frac{dV_{\text{cmax}}}{V_{\text{cmax}}}. \end{split} \tag{3}$$

The partial derivatives in Eqn 3 (l_s , l_{mc} and l_b) are functions of g_{sc} , g_m , V_{cmax} and other photosynthetic parameters. To apply Eqn 3 to finite changes in A_{max} , g_{sc} , g_m and V_{cmax} , the GM approach approximates the differentials in Eqn 3 with finite differences between two measurement points (called the 'reference' and 'comparison' points, respectively), then computes each partial derivative at both points and uses the average of the resulting two values:

Table 1. List of mathematical symbols including units

Description	Symbol	Units
Net CO ₂ assimilation rate (reference value)	$A (A_{ref})$	μmol m ⁻² s ⁻¹
Light-saturated net CO ₂ assimilation rate (reference value)	$A_{ m max}\left(A_{ m max,R} ight)$	μ mol m ⁻² s ⁻¹
A expressed as a function of 25 $^{\circ}$ C values and T	$A_{ m 25T}$	μ mol m ⁻² s ⁻¹
Ambient CO ₂ mole fraction	c_{a}	μ mol mol ⁻¹
Intercellular CO ₂ mole fraction	$c_{ m i}$	μ mol mol ⁻¹
Infinitesimal change in A (in A_{max})	$dA (dA_{\rm max})$	μ mol m ⁻² s ⁻¹
Finite change in A (in A_{max})	$\delta A \left(\delta A_{\text{max}} \right)$	μ mol m ⁻² s ⁻¹
Finite change in x_i	δx_i	varies
Initial slope of response of J to i (at 25 °C)	ϕ (ϕ_{25})	dimensionless
Total conductance to CO ₂	g	$mol m^{-2} s^{-1}$
Boundary layer conductance to CO ₂	$g_{ m bc}$	$mol \ m^{-2} \ s^{-1}$
Mesophyll conductance to CO ₂ (at 25 °C)	$g_{\rm m} \left(g_{\rm m25} \right)$	$mol m^{-2} s^{-1}$
Value of g_m at reference point (at comparison point)	$g_{m,R}(g_{m,C})$	$mol \ m^{-2} \ s^{-1}$
Stomatal conductance to CO ₂	g _{sc}	$mol \ m^{-2} \ s^{-1}$
Value of g_{sc} at reference point (at comparison point)	$g_{\rm sc,R}$ $(g_{\rm sc,C})$	$mol \ m^{-2} \ s^{-1}$
Photorespiratory CO ₂ compensation point (at 25 °C)	Γ* (Γ*25)	μ mol mol ⁻¹
Photosynthetic photon flux	i	μ mol m ⁻² s ⁻¹
Index of arbitrary variable that affects A	j	_
Potential electron transport rate	J	μ mol m ⁻² s ⁻¹
Maximum potential electron transport rate (at 25 °C)	$J_{\max} (J_{\max 25})$	μ mol m ⁻² s ⁻¹
Index for start of a subinterval of reference-comparison interval	k	_
Turnover number for RuBP carboxylation	$k_{\rm c}$	s^{-1}
Michaelis constant for RuBP carboxylation (at 25 °C)	$K_{\rm c} \left(K_{\rm c25} \right)$	μ mol mol ⁻¹
Turnover number for RuBP oxygenation	$k_{\rm o}$	s^{-1}
Michaelis constant for RuBP oxygenation (at 25 °C)	$K_{\rm o} (K_{\rm o25})$	μ mol mol $^{-1}$
Calculus-based relative limitations to A because of $g_{\rm sc}$, $g_{\rm m}$, $V_{\rm cmax}$	$l_{\rm s}, l_{\rm mc}, l_{\rm b}$	dimensionless
Averages of l_s , l_{mc} and l_b at reference and comparison points	$\overline{l}_{ m s}, \overline{l}_{ m mc}, \overline{l}_{ m b}$	dimensionless
Number of subintervals of reference-comparison interval	$n^{\iota_{\rm s},\ \iota_{ m mc},\ \iota_{ m b}}$	=
Number of variables that affect A	N	_
Ambient oxygen mole fraction	0	μ mol mol ⁻¹
Contribution of biochemical variables to a change in A	$ ho_{ m bio}$	%
Contribution of diffusional variables to a change in A	$ ho_{ m diff}$	%
Contribution of g_{m25} to a change in A	$ ho_{ m gm25}$	%
Contribution of g_{sc} to a change in A	$ ho_{ m gsc}$	%
Contribution of $J_{\text{max}}(J_{\text{max25}})$ to a change in A	$ ho_{ m Jmax} \left(ho_{ m Jmax25} ight)$	%
Contribution of K_c to a change in A	ρ_{Jmax} (ρ_{Jmax25})	%
Contribution of R_d (R_{d25}) to a change in A	$ ho_{ m Rd} \left(ho_{ m Rd25} ight)$	%
Contribution of T to a change in T (for constant 25 °C values)	$ ho_{ m T}$	%
Contribution of T to a change in T (for constant 25 °C values) Contribution of T and 25 °C values together to a change in A	$ ho_{ ext{T.25}}$	%
Contribution of V_{cmax} ($V_{\text{cmax}25}$) to a change in A	ρ_{Vcmax} (ρ_{Vcmax25})	%
Contribution of an arbitrary variable x_i to a change in A		%
Contribution of 25 °C values to a change in A (for constant T)	$ ho_{ m xj}$	%
Convexity parameter for response of J to i (at 25 °C)	ρ_{25}	dimensionless
Rate of non-photorespiratory CO ₂ release (at 25 °C)	$egin{aligned} heta_{ m j} \; (heta_{ m j25}) \ R_{ m d} \; (R_{ m d25}) \end{aligned}$	μ mol m ⁻² s ⁻¹
Leaf temperature	T	°C
Maximum carboxylation rate (at 25 °C)	÷	μ mol m $^{-2}$ s $^{-1}$
Value of V_{cmax} at reference point (at comparison point)	$V_{\text{cmax}} (V_{\text{cmax25}})$	μ mol m ⁻² s ⁻¹
Rate of triose phosphate utilization (at 25 °C)	$V_{\rm cmax,R}$ ($V_{\rm cmax,C}$)	μ mol m ⁻² s ⁻¹
Arbitrary variable that affects A	$V_{ m tpu}~(V_{ m tpu25})$	μmoi m - s · varies
Arbitrary variable that affect A except x_i , evaluated at index k	$X_{\mathbf{j}}$	
An variables that affect A except x_j , evaluated at findex κ	$\mathbf{X}_{-j,k}$	varies

$$\begin{split} \frac{A_{\text{max,R}} - A_{\text{max,C}}}{A_{\text{max,R}}} &\approx \overline{l_s} \bigg(\frac{g_{\text{sc,R}} - g_{\text{sc,C}}}{g_{\text{sc,R}}} \bigg) + \overline{l_{\text{mc}}} \bigg(\frac{g_{\text{m,R}} - g_{\text{m,C}}}{g_{\text{m,R}}} \bigg) \\ &+ \overline{l_b} \bigg(\frac{V_{\text{cmax,R}} - V_{\text{cmax,C}}}{V_{\text{cmax,R}}} \bigg), \end{split} \tag{4}$$

where the subscripts R and C denote values at the reference and comparison points, respectively, and the overbars on l_s , $l_{
m mc}$ and $l_{
m b}$ indicate averages of values at these two points. The

three groups of terms on the right-hand side of Eqn 4 are termed contributions (of g_{sc} , g_m and V_{cmax} , respectively) to the observed change in A_{max} .

Ideally, to apply Eqn 3 to finite changes, one would integrate this expression over the interval between the reference and comparison points. Equation 4 is a simple numerical approximation of that integral. However, Eqn 4 has some issues. One is that integrating Eqn 3 does not partition changes in A_{max} into contributions from changes in g_{sc} , g_{m} and $V_{\rm cmax}$; rather, it partitions changes in the natural logarithm of $A_{\rm max}$ into contributions from changes in the natural logarithms of $g_{\rm sc}, g_{\rm m}$ and $V_{\rm cmax}$. This is most easily seen by rewriting Eqn 3 as

$$d\ln A_{\text{max}} = l_{\text{s}} d\ln g_{\text{sc}} + l_{\text{mc}} d\ln g_{\text{m}} + l_{\text{b}} d\ln V_{\text{cmax}}.$$
 (3a)

Alternatively, Eqn 4 could be interpreted as an estimate of the integral of Eqn 2 (which does partition changes in $A_{\rm max}$ itself), normalized by $A_{\rm max,R}$. Even in that case, however, it is unclear whether the discrete approximation in Eqn 4 provides an unbiased estimate of that integral. Because of these issues, it is not clear whether Eqn 4 accurately partitions changes in $A_{\rm max}$ into contributions from the underlying variables.

The objectives of this paper were (1) to demonstrate that Eqn 4 does not accurately partition changes in $A_{\rm max}$, by comparing its output with numerical integrals of Eqn 2; (2) to propose a revision of the GM approach, based on numerical integration, that resolves this issue; (3) to generalize the revised approach beyond light-saturated conditions, to encompass changes in any photosynthetic variable; (4) to demonstrate the generalized approach; and (5) to provide a user-friendly computational tool for applying the generalized approach.

METHODS

In this section, we first describe a series of simulated experiments designed to compare Eqn 4 with the numerical integral of Eqn 2 (normalized by $A_{\rm max,R}$). We then describe our numerical integration approach. Finally, we describe a generalized partitioning approach based on numerical integration of Eqn 2.

Comparison of Eqn 4 with numerical integration of Eqn 2

We compared the partitioning calculated by Eqn 4 with that given by numerically integrating Eqn 2 and normalizing the result by $A_{\text{max,R}}$, in four series of simulated scenarios, each representing a change in $A_{\rm max}$ resulting from changes in $g_{\rm sc}, g_{\rm m}$ and $V_{\rm cmax}$. All scenarios shared the same reference state $(V_{\text{cmax,R}} = 150 \,\mu\text{mol m}^{-2} \,\text{s}^{-1} \text{ and } g_{\text{sc,R}} = g_{\text{m,R}} = 0.3 \,\text{mol m}^{-2} \,\text{s}^{-1}),$ but differed in the values of each variable in the comparison state; these scenarios are summarized in Table 2. In the first and second series of scenarios, $V_{\rm cmax}$ was reduced by twothirds in the comparison state (relative to the reference state), and total conductance to CO₂ was reduced by amounts ranging from 0% (no change) to 83.3%. In the third and fourth series, total conductance was reduced by half while $V_{\rm cmax}$ was reduced by amounts ranging from 0 to 83.3%. The reductions in total conductance were achieved either by reducing g_{sc} and g_{m} by equal amounts in each scenario (in series 1 and 3) or by reducing only g_{sc} (in series 2 and 4). In each case, we computed A_{max} from the carboxylation-limited version of the Farquhar et al. (1980) photosynthesis model (for details, see Supporting Information Notes S1).

Table 2. Values of variables (carboxylation capacity, $V_{\rm cmax}$; stomatal conductance to ${\rm CO_2}$, $g_{\rm sc}$; mesophyll conductance, $g_{\rm m}$; total ${\rm CO_2}$ conductance, g) used as the comparison state in the scenarios illustrated in Figs 2 and 3

Scenario		Value in comparison state					
series	Scenario	$\overline{V_{ m cmax}}$	$g_{\rm sc}$	g_{m}	g		
1	1a	50	0.25	0.25	0.125		
	1b	50	0.20	0.20	0.10		
	1c	50	0.15	0.15	0.075		
	1d	50	0.10	0.10	0.05		
	1e	50	0.05	0.05	0.025		
2	2a	50	0.214	0.3	0.125		
	2b	50	0.15	0.3	0.10		
	2c	50	0.10	0.3	0.075		
	2d	50	0.06	0.3	0.05		
	2e	50	0.027	0.3	0.025		
3	3a	125	0.1	0.3	0.075		
	3b	100	0.1	0.3	0.075		
	3c	75	0.1	0.3	0.075		
	3d	50	0.1	0.3	0.075		
	3e	25	0.1	0.3	0.075		
4	4a	125	0.15	0.15	0.075		
	4b	100	0.15	0.15	0.075		
	4c	75	0.15	0.15	0.075		
	4d	50	0.15	0.15	0.075		
	4e	25	0.15	0.15	0.075		

Reference state values were $V_{\rm cmax,R} = 150~\mu{\rm mol~m^{-2}~s^{-1}}$ and $g_{\rm sc,R} = g_{\rm m,R} = 0.30~{\rm mol~m^{-2}~s^{-1}}$ $(g = 0.15~{\rm mol~m^{-2}~s^{-1}})$.

Numerical integration of Eqn 2

For the scenarios described earlier, we estimated the integral of Eqn 2 numerically:

$$\int_{R}^{C} dA_{\text{max}} = \int_{R}^{C} \left(\frac{\partial A_{\text{max}}}{\partial g_{\text{sc}}} \right) dg_{\text{sc}} + \int_{R}^{C} \left(\frac{\partial A_{\text{max}}}{\partial g_{\text{m}}} \right) dg_{\text{m}} + \int_{R}^{C} \left(\frac{\partial A_{\text{max}}}{\partial V_{\text{cmax}}} \right) dV_{\text{cmax}},$$
(5)

where the limits of integration ('R' and 'C') refer to the reference and comparison points, respectively. The three integrals on the right-hand side of Eqn 5 represent the contributions of $g_{\rm sc}$, $g_{\rm m}$ and $V_{\rm cmax}$, respectively, to the change in A between the reference and comparison points. These integrals can be estimated numerically by approximating the differentials and derivatives therein as finite differences and ratios of finite differences, respectively. For example, for the term in Eqn 2 involving $g_{\rm sc}$:

$$\int_{R}^{C} \left(\frac{\partial A_{\text{max}}}{\partial g_{\text{sc}}} \right) dg_{\text{sc}} \approx \sum_{k=0}^{n-1} \left(\frac{\Delta A_{\text{max}}}{\Delta g_{\text{sc}}} \bigg|_{g_{\text{m}}, V_{\text{cmax}}}^{k,k+1} \right) \cdot \Delta g_{\text{sc}} = \sum_{k=0}^{n-1} \left(\Delta A_{\text{max}} \bigg|_{g_{\text{m}}, V_{\text{cmax}}}^{k,k+1} \right), \tag{5a}$$

where the summation occurs over n equal subdivisions of the interval between the reference and comparison points (k=0 and n, respectively); the superscript 'k, k+1' means that the changes $\Delta A_{\rm max}$ and $\Delta g_{\rm sc}$ are computed between indices k and k+1; and the subscript ' $g_{\rm m}, V_{\rm cmax}$ ' indicates that $\Delta A_{\rm max}$ is computed by changing $g_{\rm sc}$ while holding $g_{\rm m}$ and $V_{\rm cmax}$ constant.

This change in $A_{\rm max}$ could be estimated by computing $\partial A_{\rm max}/\partial g_{\rm sc}$ analytically and multiplying it by a small finite increment in $g_{\rm sc}$, but it can be computed more easily and directly by simply changing $g_{\rm sc}$ in the photosynthesis model. For example, Eqn 5 becomes

$$= \sum_{k=0}^{n-1} [A_{\max}(g_{sc,k+1}, g_{m,k}, V_{c\max,k}) - A_{\max}(g_{sc,k}, g_{m,k}, V_{c\max,k})], \quad (5b)$$

where $A_{\rm max}(g_{\rm sc,k}, g_{\rm m,k}, V_{\rm cmax,k})$ refers to the carboxylationlimited form of the photosynthesis model, evaluated at the values of $g_{\rm sc}$, $g_{\rm m}$ and $V_{\rm cmax}$ indicated by the index k. An example of the application of Eqn 5 is described in Table 3 and illustrated in Fig. 1. This approach avoids partial derivatives altogether, which greatly simplifies its generalization to other variables (as shown later).

For clarity, we adopt the following simplified and generalized notation:

$$\left[\delta A |\delta x_i|_{l_i}^{k+1} \equiv A\left(x_{i,k+1}, \mathbf{x}_{\neg i,k}\right) - A\left(x_{i,k}, \mathbf{x}_{\neg i,k}\right), \tag{6}$$

where \mathbf{x}_{-j} means 'all variables other than x_j ' (' \neg ' means 'not'). In the notation on the left hand side of Eqn 6, variables that appear after the vertical bar are allowed to change, and all other variables are held constant. Equation 5 thus becomes

$$\int_{R}^{C} \left(\frac{\partial A_{\text{max}}}{\partial g_{\text{sc}}} \right) dg_{\text{sc}} \approx \sum_{k=0}^{n-1} \left[\delta A_{\text{max}} | \delta g_{\text{sc}} \right]_{k}^{k+1}.$$
 (5c)

The other terms in Eqn 2 (those involving g_m and V_{cmax}) are integrated in the same manner as shown in Eqn 5c.

The numerical integration represented by Eqn 5 requires an assumption about how g_{sc} , g_m and V_{cmax} vary across the interval – that is, about the 'paths' taken by these variables between the reference and comparison points. The simplest

assumption, which we adopt here, is that the variables change at a uniform rate (i.e., linearly with 'time', if the interval is understood to represent a period of time). Thus, $g_{\text{sc,k}} = g_{\text{sc,R}} + k \cdot (g_{\text{sc,C}} - g_{\text{sc,R}})/n$, and likewise for $g_{\text{m,k}}$ and $V_{\text{cmax,k}}$. We note that GM also assumes paths for each variable: Eqn 4 represents an approximate integral of Eqn 2 (cf. their Eqns 6 and 8), and Eqn 2 cannot be integrated without specifying such paths. The key difference is that our approach clearly and explicitly identifies these paths.

Note that Eqn 5 gives contributions with the opposite sign to those calculated by Eqn 4 (because Eqn 5 treats the reference point as the lower integration bound), so when comparing these equations, we multiplied the output of Eqn 5 by minus 1. However, the generalized approach described later retains the sign convention of Eqn 5.

Generalization of the revised approach

We propose a generalized approach to partitioning changes in A into contributions from the underlying variables. In this new approach, Eqn 2 is numerically integrated across the interval between reference and comparison points and the resulting contributions are expressed as percentages of the reference value of A (A_{ref}). The contribution from a variable x_i to a change in A is defined as

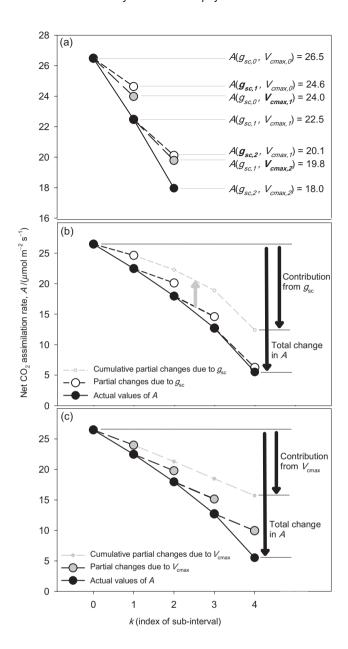
$$\rho_{x_{j}} = \frac{100}{A_{\text{ref}}} \cdot \sum_{k=0}^{n-1} [\delta A | \delta x_{j}]_{k}^{k+1}.$$
 (7)

Because this approach does not use partial derivatives, but instead computes partial changes in A directly from the photosynthesis model, it is easily generalized to arbitrary conditions (such as sub-saturating photosynthetic photon flux, PPF) and to variables other than $g_{\rm sc}$, $g_{\rm m}$ and $V_{\rm cmax}$. The generalization is most clearly presented by expressing A as a function of many variables:

Table 3. Example application (also illustrated in Fig. 1) of the method presented in this paper for partitioning changes in A (μ mol m⁻² s⁻¹) into contributions from the underlying variables

		Values						
Description	Symbol or expression	(Ref)				(Comp)	Total change	
Index of sub-interval	k	0	1	2	3	4		
$g_{\rm sc}$ at start of sub-interval	$g_{\mathrm{sc,k}}$	0.3	0.23	0.16	0.1	0.03		
$V_{\rm cmax}$ at start of sub-interval	$V_{ m cmax,k}$	150	125	100	75	50		
A at start of sub-interval	$A(g_{\rm sc,k}, V_{\rm cmax,k})$	26.5	22.5	18.0	12.7	5.5		
A at new g_{sc}	$A(g_{ m sc,k+1},V_{ m cmax,k})$	24.6	20.1	14.6	6.2			
A at new $V_{\rm cmax}$	$A(g_{\mathrm{sc,k}}, V_{\mathrm{cmax,k+1}})$	24.0	19.8	15.1	10.0			
Change in A because of g_{sc}	$A(g_{\text{sc,k+1}}, V_{\text{cmax,k}}) - A(g_{\text{sc,k}}, V_{\text{cmax,k}})$	-1.9	-2.4	-3.4	-6.5		-14.1	
Change in A because of $V_{\rm cmax}$	$A(g_{\text{sc,k}}, V_{\text{cmax,k+1}}) - A(g_{\text{sc,k}}, V_{\text{cmax,k}})$	-2.5	-2.7	-2.8	-2.8		-10.8	
Actual change in A	$A(g_{\text{sc,k+1}}, V_{\text{cmax,k+1}}) - A(g_{\text{sc,k}}, V_{\text{cmax,k}})$	-4	-4.5	-5.2	-7.2		-21	
% contribution from g_{sc}	$100 \cdot \text{(change due to } g_{\text{sc}})/\text{(reference value of } A)$						-53.1%	
% contribution from $V_{ m cmax}$	$100 \cdot \text{(change due to } V_{\text{cmax}})/\text{(reference value of } A)$						-40.7%	

In this example, it is assumed that only two variables (g_{sc} and V_{cmax}) change between the reference ('ref') and comparison ('comp') points, from 0.3 to 0.027 mol m⁻² s⁻¹ (g_{sc}) and from 150 to 50 μ mol m⁻² s⁻¹ (V_{cmax}). The interval between those points is divided into n sub-intervals (n = 4 in this example), whose starting and ending points are indicated by the index k. Partial changes in A between each successive sub-interval are calculated based on the corresponding changes in g_{sc} and V_{cmax} , and the contributions are calculated based on the sums of these changes, as shown. We recommend using n = 1000.



$$A = A \begin{pmatrix} g_{sc}, g_{bc}, g_{m}, V_{cmax}, J_{max}, V_{tpu}, \\ R_{d}, K_{c}, \Gamma_{*}, \phi, \theta_{j}, c_{a}, O, i \end{pmatrix},$$
(8)

where g_{bc} is boundary layer conductance to CO₂; J_{max} is maximum potential electron transport rate; V_{tpu} is triose phosphate utilization (TPU) rate; R_d is the rate of nonphotorespiratory CO₂ release; K_c and K_o are the Michaelis constants for ribulose-1,5-bisphosphate (RuBP) carboxylation and oxygenation, respectively; Γ_* is the photorespiratory CO₂ compensation point; i is PPF; ϕ is the initial slope of the response of potential electron transport rate (J) to i; θ_i is a dimensionless convexity parameter for the response of J to i; c_a is ambient CO₂ concentration; and O is ambient O₂ concentration. (Note that Γ_* is not independent of K_c , K_o and O in the original Farquhar et al. (1980) model, but is given by

Figure 1. Example illustrating the sequence of partial and total changes in A because of changes in g_{sc} and V_{cmax} , calculated for sub-intervals of a total interval between reference and comparison conditions. In this example, the reference-to-comparison interval is divided into four sub-intervals, denoted by the index k; the reference and comparison points correspond to k = 0 and k = 4, respectively. The values shown in this figure correspond to the example detailed in Table 3. (a) Example showing values of A calculated at the first three points in the interval (k = 0, 1 and 2), and the corresponding values of A calculated by changing only g_{sc} (white symbols, short-dashed lines), or by changing only $V_{\rm cmax}$ (grey symbols, long-dashed lines) across each successive sub-interval. Note that the initial condition for each of these 'partial changes' in A is the actual value of A at the start of the sub-interval. (b) Values of A (solid symbols, solid line), partial changes in A because of g_{sc} (large white symbols, short-dashed black line), and an imaginary sequence of A that would result from accumulating only the partial changes in A because of g_{sc} (small white symbols, grey dashed line). (c) Values of A (solid symbols, solid line), partial changes in A because of $V_{\rm cmax}$ (large grey symbols, long-dashed black line), and an imaginary sequence of A that would result from accumulating only the partial changes in A because of $V_{\rm cmax}$ (small grey symbols, grey dashed line). In (a), values of A at each point are given for cross-referencing with Table 3. In (b), an upward grey arrow is shown for one of the partial changes, to illustrate that the segments in the grey dashed line correspond to the partial changes. In (b) and (c), the sum of partial changes in g_{sc} and V_{cmax} , respectively, are shown by the shorter black arrows ('contributions'), and the total changes in A are shown by the longer black arrows.

 $\Gamma_* = O \ K_c \ k_o / (2K_o \ k_c)$, where k_o and k_c are the Rubisco turnover numbers for RuBP oxygenation and carboxylation, respectively. In practice, investigators often treat Γ_* as an empirical parameter. Users preferring the initial formulation should replace Γ_* with k_c and k_o in Eqn 8. Note also that Eqn 8 omits the influence of the evaporative gradient, which, together with g_{sc} and g_{bc} , determines transpiration rate; the latter in turn affects A via ternary interactions between H_2O and CO_2 .)

An alternative formulation that separates the effects of temperature (T) and 25 °C values of T-dependent variables is shown in Eqn. 9.

$$A_{25T} = A_{25T} \begin{pmatrix} g_{sc}, g_{bc}, g_{m25}, V_{c \max 25}, J_{\max 25}, V_{tpu25}, R_{d25}, \\ K_{c25}, K_{o25}, \Gamma_{*25}, \phi_{25}, \theta_{j25}, c_{a25}, pO_2, i, T \end{pmatrix}.$$
(9)

The contribution from a change in the 25 °C value (x_{j25}) of a variable x_j , independent of changes in T, can then be defined as

$$\rho_{xj25} = \frac{100}{A_{\text{ref}}} \cdot \sum_{k=0}^{n-1} [\delta A_{25T} | \delta x_{j25}]_{k}^{k+1}, \tag{10}$$

and the total contribution from changes in all 25 °C values per se is

$$\rho_{25} = \frac{100}{A_{\text{ref}}} \cdot \sum_{k=0}^{n-1} \left[\frac{\delta A_{25T} | \delta g_{\text{m25}}, \delta V_{\text{cmax 25}}, \delta J_{\text{max 25}}, \delta V_{\text{tpu25}}, \delta R_{\text{d25}},}{\delta K_{\text{c25}}, \delta K_{\text{o25}}, \delta \Gamma_{*25}, \delta \phi_{25}, \delta \theta_{j25}} \right].$$
(11)

The contribution from T per se (ρ_T) , independent of changes in 25 °C values, is

$$\rho_{\rm T} = \frac{100}{A_{\rm ref}} \cdot \sum_{k=0}^{\rm n-1} [\delta A_{\rm 25T} | \delta T]_{\rm k}^{\rm k+1}. \tag{12}$$

A distinct notation ($A_{25\mathrm{T}}$) is used to represent the functional form of A given in Eqn 9 and relevant to Eqns 12 and 11, to clarify that it is the 25 °C values of T-dependent variables, and not their temperature-adjusted values, that are held constant when computing ρ_{T} . An *overall* contribution from temperature, including both the direct effect of T and changes in 25 °C values, can be defined as

$$\rho_{\text{T},25} = \frac{100}{A_{\text{ref}}} \cdot \sum_{k=0}^{n-1} \left[\delta A | \delta g_{\text{m}}, \delta V_{\text{c max}}, \delta J_{\text{max}}, \delta V_{\text{tpu}}, \right]_{k}^{k+1}.$$
(13)

Similarly, the total contribution from diffusional conductances $(g_{sc}, g_m \text{ and } g_{bc})$ is

$$\rho_{\text{diff}} = \frac{100}{A_{\text{ref}}} \cdot \sum_{k=0}^{n-1} [\delta A | \delta g_{\text{sc}}, \delta g_{\text{bc}}, \delta g_{\text{m}}]_{k}^{k+1}, \tag{14}$$

and the total contribution from variables that involve the biochemistry of photosynthesis is

$$\rho_{\text{bio}} = \frac{100}{A_{\text{ref}}} \cdot \sum_{k=0}^{n-1} \left[\frac{\delta A |\delta V_{\text{cmax}}, \delta J_{\text{max}}, \delta V_{\text{tpu}}, \delta R_{\text{d}},}{\delta K_{\text{c}}, \delta K_{\text{o}}, \delta K_{\text{o}}, \delta \Gamma_{*}, \delta \phi, \delta \theta_{\text{j}}, \delta i} \right]_{k}^{k+1}.$$
(15)

We include the effect of PPF (i) in ρ_{bio} because the direct effect of i on A occurs via J, which is usually viewed as a biochemical variable.

Demonstration of the generalized approach

We applied Eqns 7 and 12–15 to field measurements of leaf gas exchange in olive ($Olea\ europaea\ L$.) trees, performed in 2002 and partially published in 2007 (Diaz-Espejo $et\ al.$ 2007). We measured responses of A to intercellular CO_2 mole fraction (c_i) and diurnal cycles of leaf gas exchange at two canopy positions (east- and west-facing) in four trees under two watering treatments (well-watered and water-stressed), in two seasons (April and August) (for details, see Diaz-Espejo $et\ al.$ 2007). Both positions had similar daily-integrated PPF, but different patterns of air humidity, temperature and time of peak PPF.

All measurements used a portable photosynthesis system (LI-6400, Li-Cor, Lincoln, NE, USA) with a 2×3 cm broadleaf chamber and an integrated light source (LI-6400-02B; Li-Cor). We estimated $V_{\rm cmax}$, $J_{\rm max}$, TPU and $g_{\rm m}$ by fitting the photosynthesis model of Farquhar *et al.* (1980) to A versus $c_{\rm i}$ curves, following the curve-fitting method proposed by Ethier & Livingston (2004). The curves were performed under saturating PPF (1600 μ mol m⁻² s⁻¹) and constant leaf temperature (20 °C in April and 25 °C in August) by changing the CO₂ concentration of inlet air in 11 steps from 50 to 1400 μ mol mol⁻¹ (see Díaz-Espejo *et al.* 2006 for details). Curves were measured for six leaves per treatment, per

canopy position in April 2002, and four leaves per treatment and position in August 2002. Diurnal cycles of A and g_s in situ (seven measurements per day) were measured in August on 12 leaves (three per tree × four trees) per treatment and canopy position, and the results were averaged within each treatment/position pair. Temperature dependencies of photosynthetic parameters were calculated according to Bernacchi et al. (2002), using parameters specifically determined for olive leaves (Díaz-Espejo et al. 2006) and modified to account for the effect of g_m (for details, see Supporting Information Notes S1).

We defined the reference point as the point at which A was greatest; this was always April for seasonal changes, but it varied among treatments for diurnal cycles.

Numerical procedures

We implemented the calculations described earlier using worksheet and user-defined functions and Visual Basic for Applications (VBA) subroutines in a Microsoft Excel 2010 spreadsheet (Microsoft Corp., Redmond, WA, USA), which is included as Supporting Information Notes S2, and is available from the authors upon request.

RESULTS

Choice of number of steps for numerical integration

To quantify the trade-off between speed and accuracy in numerical integration of Eqn 2, we computed the sum of contributions to seasonal changes in A for all variables in the four field treatments described earlier, for a range of values of n (the number of numerical integration steps). We estimated the true value of each integral as its numerical integral using n = 30,000. The percentage error of numerical integration declined as the inverse of n ($\ln |\%$ error $| = -0.98 \ln |n| + 3.32$; adjusted $r^2 = 0.96$, P < 0.0001, degrees of freedom = 74; not shown). For n = 1000, the error was less than 0.07% across datasets, and the calculations took 1.3 s per comparison point on a modern personal computer with many other programs running. We conclude that numerical integration with n = 1000 is adequate and feasible, and we used n = 1000 for all calculations presented here.

Comparison of Eqn 4 (the GM approach) with numerical integration of Eqn 2

Equation 4 systematically underestimated the percentage contributions of $V_{\rm cmax}$ and $g_{\rm sc}$ to simulated changes in $A_{\rm max}$ (Figs 2 & 3). The degree of underestimation differed if a given decrease in total CO₂ conductance was effected by reducing both $g_{\rm sc}$ and $g_{\rm m}$ (scenario series 1 and 3) or by reducing only $g_{\rm sc}$ (scenario series 2 and 4) (e.g. compare Fig. 2a,c and Fig. 2b,d, or Fig. 3a,c and Fig. 3b,d). Furthermore, the contributions calculated using Eqn 4 were nonlinearly related to those computed by numerical integration. For example, Eqn 4 underestimated the $g_{\rm sc}$ contribution by

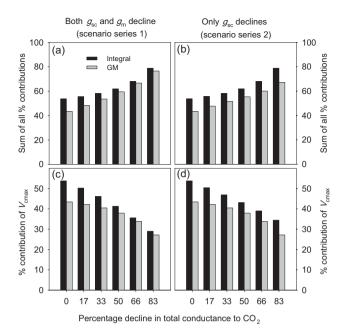


Figure 2. Comparison of contributions to changes in $A_{\rm max}$ computed using the GM approach (Eqn 4) (grey bars, 'GM') or by numerically integrating Eqn 2 (black bars, 'integral'), for different scenarios in which total CO₂ conductance was reduced by different percentages (horizontal axis), either by reducing both stomatal conductance ($g_{\rm sc}$) and mesophyll conductance ($g_{\rm m}$) equally (panels a and c; scenario series 1) or by reducing only $g_{\rm sc}$ (panels b and d; scenario series 2). (a,b) The sum of contributions from all variables combined. (c,d) The contribution from $V_{\rm cmax}$. Scenarios are described in the text and summarized in Table 2.

24.5% when $V_{\rm cmax}$ was identical in the reference and comparison states, but by 8.1% when $V_{\rm cmax}$ decreased by 83% in the comparison state (Fig. 3c). These results demonstrate that Eqn 4 is a biased tool for partitioning changes in $A_{\rm max}$.

Demonstration of the generalized approach for seasonal changes

Figure 4a shows the contributions estimated using the new approach. The corresponding 25 °C values of $V_{\rm cmax}$, $J_{\rm max}$, $R_{\rm d}$ and $g_{\rm m}$, and the observed values of $g_{\rm sc}$, are given in Table 4.

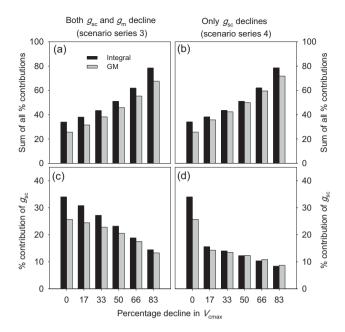


Figure 3. Comparison of contributions to changes in $A_{\rm max}$ computed using the GM approach (Eqn 4) (grey bars, 'GM') or by numerically integrating Eqn 2 (black bars, 'integral'), for different scenarios in which carboxylation capacity ($V_{\rm cmax}$) was reduced by different percentages (horizontal axis), and total CO₂ conductance was reduced by half, either by reducing both stomatal conductance ($g_{\rm sc}$) and mesophyll conductance ($g_{\rm m}$) equally (panels a and c; scenario series 3) or by reducing only $g_{\rm sc}$ (panels b and d; scenario series 4). (a,b) The sum of contributions from all variables combined. (c,d) The contribution from $g_{\rm sc}$. Scenarios are described in the text and summarized in Table 2.

Total relative changes in A were greater under water stress, mainly because of greater stomatal contributions: $\rho_{\rm gsc}$ was positive in well-watered conditions (+6.7 to +11.8%) because $g_{\rm sc}$ increased from April to August (Table 4), whereas $\rho_{\rm gsc}$ was negative under water stress (-14.6 to -23.9%; red bars in Fig. 4a). The contribution from $g_{\rm m}$ (including the effects of changes in its 25 °C value and changes in temperature) was negative in all treatments, but was generally smaller than the stomatal contribution, ranging from -3.0 to -9.0%. $\rho_{\rm Vcmax}$ was similar in magnitude to $\rho_{\rm gsc}$, but was always negative (-11.3 to -24.7%).

	Well-watered				Water-stressed			
	East-facing		West-facing		East-facing		West-facing	
Variable	Ref	Comp	Ref	Comp	Ref	Comp	Ref	Comp
$V_{\rm cmax25}$	160.3	74.0	168.3	75.8	169.0	83.2	168.6	49.9
$J_{ m max25}$	186.8	212.2	223.6	213.4	292.3	214.5	290.2	215.6
$R_{\rm d25}$	1.2	3.0	1.6	4.0	2.1	6.0	2.8	5.0
g _{m25}	0.331	0.163	0.337	0.204	0.335	0.149	0.371	0.117
$g_{\rm sc}$	0.135	0.156	0.117	0.166	0.154	0.093	0.179	0.058

For photosynthetic variables not listed here, values were not independently measured across treatments, and assumed values are given in the Supporting Information Notes S1. Units for V_{cmax25} , J_{max25} and R_{d25} are μ mol m⁻² s⁻¹, and units for g_{m25} and g_{sc} are mol m⁻² s⁻¹.

Table 4. Values of photosynthetic variables in April (reference point, 'ref') and August (comparison point, 'comp') in the four experimental treatments (well-watered versus water-stressed, and east- versus west-facing sections of an olive canopy) used to demonstrate the generalized approach for seasonal changes

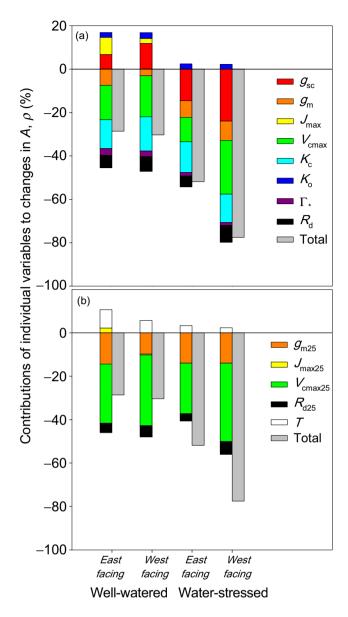


Figure 4. Contributions of individual variables (ρ) to seasonal changes in light-saturated net CO_2 assimilation rate (A_{max}) across four treatments as shown below the horizontal axis, with the reference point taken as spring measurements (April) and the comparison point taken as late summer measurements (August). (a) Contributions from total changes in each variable, including effects of temperature and changes in values of variables at 25 °C (for g_m , J_{max} , V_{cmax} and R_d). (b) Contributions from changes in values at 25 °C (g_{m25} , J_{max25} , V_{cmax25} and R_{d25}), and the total contribution from changes in temperature (T) calculated while holding 25 °C values constant. 'Total' refers to the total relative change in A, which equals the sum of contributions for all variables).

Substantial fractions of the total seasonal changes in A were due to changes in variables other than g_{sc} , g_m and V_{cmax} . For example, the contributions from K_c and R_d ranged from -13.0 to -15.8% (for K_c) and from -5.1 to -7.8% (for R_d). The contribution from J_{max} was positive in the well-watered

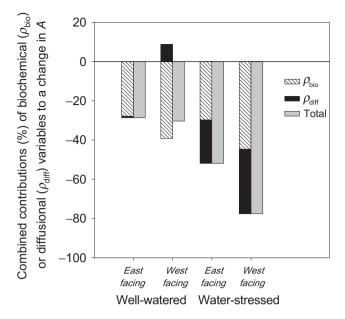


Figure 5. Contributions of groups of variables (ρ) to seasonal changes in light-saturated net CO_2 assimilation rate (A_{max}) across four treatments, with the reference point taken as spring measurements (April) and the comparison point taken as late summer measurements (August). 'bio', white bars with hatching: total contribution from changes in variables of photosynthetic biochemistry (J_{max} , V_{cmax} , K_{c} , K_{o} , Γ_{*} , R_{d} and i). 'diff', black bars: total contribution from changes in diffusional variables (g_{sc} and $g_{\rm m}$). Other variables included in $\rho_{\rm bio}$ and $\rho_{\rm diff}$ in their respective definitions in the text were assumed or observed to be constant among treatments. 'Total', grey bars: the total relative change in A_{max} , which equals the sum of contributions for all variables.

treatment (+2.3 to +7.9%), but 0 under water stress (because photosynthesis was carboxylation-limited in both seasons).

Figure 4b shows the contributions from temperaturerelated variables separated into the effects of changes in T per se (calculated while holding 25 °C values constant) and changes in 25 °C values. The direct effect of T was small, but positive in each case, ranging from +2.4 to +8.5% (because measurement T was slightly higher in August), whereas the total contribution from 25 °C values was large and negative, ranging from -40.6 to -56.0%. This was largely driven by large seasonal decreases in the value of $V_{\rm cmax}$ at 25 °C, but also by increases in the value of $R_{\rm d}$ at 25 °C.

Figure 5 shows the combined contributions from seasonal changes in all diffusional variables (ρ_{diff}) and all biochemical variables ($\rho_{\rm bio}$). $\rho_{\rm diff}$ was negligible in the east-facing wellwatered treatment, positive in the west-facing well-watered treatment, and large and negative in both water-stressed treatments. By contrast, ρ_{bio} was large and negative, and greater in magnitude than p_{diff} , in all cases.

Demonstration of the generalized approach for diurnal changes

The diurnal trends in A, g_{sc} , g_m , T and i observed in August are shown in Fig. 6, and the associated contributions are shown in

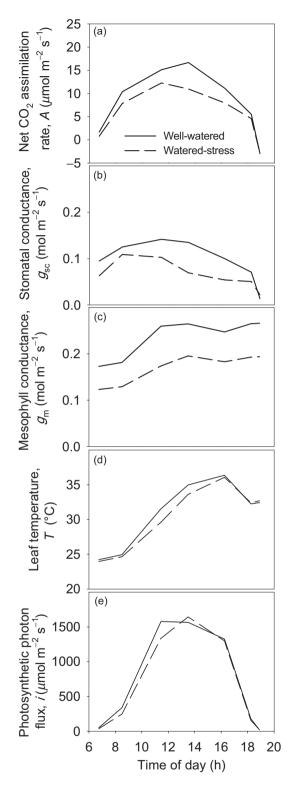


Figure 6. Diurnal changes in (a) net assimilation rate; (b) stomatal conductance; (c) mesophyll conductance; (d) temperature; and (e) photosynthetic photon flux, in well-watered (solid lines) and water-stressed (dashed lines) treatments in olive.

Figs 7 and 8 for well-watered and water-stressed conditions, respectively. In each figure, $\rho_{\rm diff}$ and $\rho_{\rm bio}$ are presented in panel (a) and broken down into individual variables in panels (b) – (e). Low PPF caused $\rho_{\rm bio}$ to dominate in the morning, whereas $\rho_{\rm bio}$ and $\rho_{\rm diff}$ were similar late in the day (Figs 7a & 8a). Late-day diffusional suppression of A was entirely due to $g_{\rm sc}$ (Figs 7b & 8b). The effect of biochemical variables other than PPF was small across the day (Figs 7c & 8c). This reflected a fine balance among the effects of several variables, particularly between $V_{\rm cmax}$ and $K_{\rm c}$: $\rho_{\rm Vcmax}$ was negative in the morning and positive in the afternoon (Figs 7d & 8d), but this was opposed by a similar, but reverse trend in $\rho_{\rm Kc}$ (Figs 7e & 8e).

DISCUSSION

Comparison of our approach with that of Grassi & Magnani (2005)

The approach presented earlier for partitioning changes in A into components because of the underlying variables uses numerical integration (e.g. Eqn 5) rather than discrete approximations of differentials and partial derivatives (cf. Eqn 4). Our approach has two major advantages: it avoids the bias caused by the discrete approximations in Eqn 4 (Figs 2 & 3), and by avoiding the need to compute partial derivatives for each variable and relying instead on substitution in the photosynthesis model, our approach is easily extended to encompass effects of changes in any photosynthetic variable, under any conditions. This extension also allows the total contribution of changes in temperature to be calculated, including both the direct effect of T and the effect of changes in 25 °C values. This total effect may be more relevant for quantifying the adaptive significance of seasonal changes in photosynthetic parameters such as $V_{\rm cmax}$, because selection acts more directly on temperature-adjusted parameters than on their 25 °C values.

A constraint of the new approach is that it requires a fully parameterized photosynthesis model. However, only three additional parameters ($J_{\rm max}$, $R_{\rm d}$ and $g_{\rm bc}$) are required beyond those needed to apply the method of Grassi & Magnani (2005), and these three parameters are typically measured or estimated in the same procedures used to estimate $V_{\rm cmax}$. Both methods rely on calculations based on a model (to compute its derivatives in the GM approach or to compute small changes by direct substitution in our approach), so both methods are only meaningful to the extent that the model adequately describes how each variable affects A.

The terminology associated with photosynthetic limitations analysis is sometimes ambiguous. For example, 'limitations' is used to describe both the contributions of changes in variables to a change in A, and the extent to which A is limited by those variables at a given condition, irrespective of comparisons with any other measured condition (e.g. Farquhar & Sharkey 1982). To avoid ambiguity, we recommend using the term 'contributions' to describe the quantities calculated by our method.

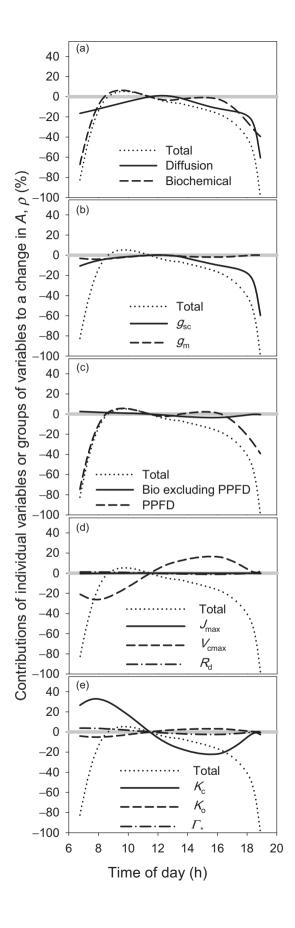


Figure 7. Contributions of individual variables and groups of variables to diurnal changes in net CO2 assimilation rate (A) under well-watered conditions in olive, with the reference point taken as the time of day at which A was greatest. In each panel, the total relative change in A, which equals the sum of contributions for all variables, is shown with dotted lines ('Total') for reference. Grey horizontal lines represent zero on the vertical axis. (a) Total contribution from diffusional variables (ρ_{diff} , solid line) and biochemical variables (ρ_{bio} , dashed line). (b) Components of ρ_{diff} : $\rho_{\rm gsc}$ (solid line) and $\rho_{\rm gm}$ (dashed line). (c-e) Components of $\rho_{\rm bio}$: (c) All components of ρ_{bio} except the component because of photosynthetic photon flux (PPF), ρ_i ('bio excluding PPF', solid line), and ρ_i ('PPF', dashed line). (d) Components of ρ_{bio} with flux dimensions: ρ_{Jmax} (solid line), ρ_{Vcmax} (dashed line) and ρ_{Rd} (dash-dot line). (e) Kinetic components of ρ_{bio} : ρ_{Kc} (solid line), ρ_{Ko} (dashed line) and ρ_{Γ^*} (dash-dot line).

Issue of path dependence

Jones (1985) noted that explicit integration of Eqn 2, which is the basis of our method, 'generally will not be feasible, because of a lack of detailed information both on the function A(...) and on the actual path followed between [the reference and comparison conditions].' The first challenge has since been overcome by the universal adoption of the Farquhar et al. (1980) photosynthesis model, combined with the ubiquity of gas exchange systems that allow its parameters to be estimated. We argue that the second challenge is not especially relevant to the questions that most investigators ask when they seek to partition changes in A. Although a path of change in each variable is indeed required to compute the integral, it is doubtful that most users are interested in how the variable's actual path would affect its calculated contribution to a change in A. Our method adopts the simplest assumption, which is that each variable changes at a uniform rate between the reference and comparison points. This allows contributions to be calculated in a standardized and unambiguous way.

One issue that can arise with this approach is that the results depend on whether the effect of stomata is expressed as a conductance (g_{sc}) or a resistance $(r_{sc} = g_{sc}^{-1})$, because linear changes in g_{sc} and r_{sc} give different sequences of physiological states. For example, suppose g_{sc} changes from 0.5 to 0.1 mol m⁻² s⁻¹. The value of g_{sc} at the midpoint of the interval is 0.3 if g_{sc} changes linearly, but 0.17 if r_{sc} changes linearly (from 2 to 10 m² s mol⁻¹). However, most modern work in this field uses stomatal conductance rather than resistance, perhaps because both g_{sc} and V_{cmax} are positively related to limiting photosynthetic resources (the rate of water loss, and the photosynthetic nitrogen allocated to Rubisco, respectively), so the issue probably has little impact. At any rate, a similar issue would also arise if r_{sc} were used in place of g_{sc} in the GM method – indeed, changes in A cannot be unambiguously partitioned into changes due to the underlying variables without specifying paths for those variables, because Eqns 1 and 2 cannot be integrated without specifying paths. A strength of our approach is that it clearly and explicitly specifies these paths.

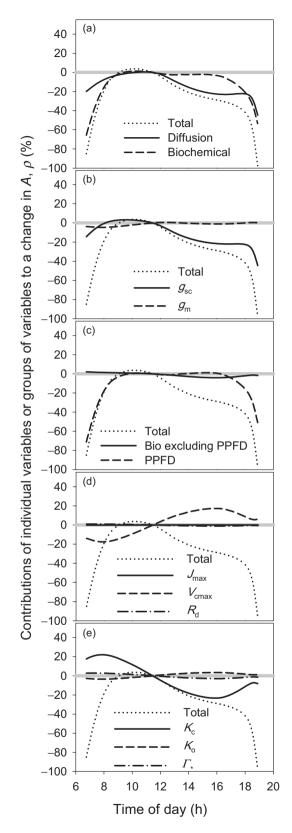


Figure 8. Contributions to diurnal changes in net CO₂ assimilation rate (A) in olive (as in Fig. 7, but for measurements made under water-stressed conditions).

Conclusion

The ubiquity of powerful desktop computing and an easily parameterized biophysical photosynthesis model obviate the approximations that were necessary in the past to partition changes in A. The direct, computationally intensive approach proposed here is now practical. We suggest further that our method is preferable to alternative methods that attempt to partition changes in A by coarsely approximating the integral of its exact differential (Eqn 1) using partial derivatives which yields biased and ambiguous partitioning.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article at the publisher's web-site:

Note S1. Description of the photosynthesis model used in this study.

Note S2. Spreadsheet implementing the calculations described in this study.